RECONSTRUCTION TOMOGRAPHY USING CHORD-SEGMENT-INVERSION TECHNIQUE

by

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NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1987

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

R K. JARWAL

to the

NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1987

NETP-1987-M-JAR-REC

CERTIFICATE

This is to certify that this work on "Reconstruction Tomography Using Chord-Segment-Inversion Technique" by Mr. R.K. JARWAL has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.

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March, 1987

ACKNOWLEDGEMENTS

I am indebted to my guides, Mr. Prabhat Munshi and Dr.R.K.S.Rathore for their best guidance and encouragement provided throughout the course of study in making this thesis possible. The valuable advice of Prof.K.Sri Ram, Prof. A.Sengupta, Dr. M.S. Kalra and Dr.I.D.Dhariyal is gratefully acknowledged.

The help provided by Major V.K.Bhatia, Mr.Manoj Kumar, Mr. Sunil Kumar and other colleagues will be long remembered.

The comradeship and love of "ALL-VILLAINS" gang, will be cherished for a long time.

A big thanks to Messrs V.S.Tomar, S. Yadav, S.S.Pathak, Gopal Krishna of Nuclear Engineering and Technology (NET) Program and Mr. U.S. Mishra for typing the thesis skillfully.

Finally many-many thanks are for my Parents, Brothers, Raju, Gautam, Pusha and Baba-Mawasi, not forgetting Banti, their love and encouragement during my stay away from home, that kept me going.

March 2, 1987. -R.K.JARWAL

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ABSTRACT

Computerized Tomography (CT) has been demonstrated to be a good technique for measuring point-density (void-fraction) in two-phase flow systems. Recently, improvements have been suggested regarding the choice of filter functions in CT methods. These methods are based, essentially, on the discrete implementation of the Radon Inversion Formulae which is widely used in the medical imaging area. Such methods do not require any information, a priori, regarding the distribution of the density (or the void-fraction).

A very simple method involving the tomographic chord-segment inversion, has been developed and tested for two-phase flows having radially symmetric density distributions. This method is much simpler and consumes less CPU time relative to the more general methods of tomographic reconstruction. For test functions the reconstructed density distributions are almost exact. For an air-water bubbly flow data the reconstructed values have a maximum deviation of $\pm$ 0.03 g/cm^3. The range of investigation of the air-water flow data was 0.6 - 0.9 g/cm^3, i.e. void-fraction range of 40% to 10%. These results are comparable to the results obtained by the more general methods based on the Radon Inversion Formulae.

CHAPTER-1

INTRODUCTION

Computer Aided Tomography (CAT) is being widely used in the medical area for the diagnosis of various cancerous tissues. The methodology incorporates scanning of the patient with gamma rays using appropriate tomographic algorithms to reconstruct the density distribution of tissues [1] . A basic form of CAT was used in Japan in 1946 named "Rotation Radiography". In this method the patient was placed on a rotatographic table, X-ray tube and film were rotated around the patient from 0°-360° while the pictures were taken and collected information regarding various cross sections in the range 0°-360°.

This concept of measuring density distribution was first investigated by Schlosser et al [2] for a two-phase air-water system. The results obtained in void fraction/ density measurements have been summarized by Kulacki et al [3]. The technique for measuring density distribution has great significance because accurate measurements of density for various flow systems and components in nuclear systems facilitates the computation of heat transfer rates. This information is vital from the reactor safety view-point because it helps in predicting core burnout etc. This technique can be applied in other fields like chemical

industries, food-processing and several other research areas.

The various reconstruction methods can be broadly classified in the following categories;

(a) Series Expansion Methods
(b) Transform Methods.

In the Series Expansion Methods (SEMs) the pixel-wise distribution of the function (under-investigation) is assumed and then suitable iterative and noniterative procedures are applied to achieve the reconstruction of the function in the region of interest [4]. The iterative SEMs are Algebraic Reconstruction Technique (ART), Simultaneous Iterative Reconstruction Technique (SIRT), etc. The non-iterative SEMs are Angular Harmonic Decomposition (AHD) and Polynomial Decomposition (PD).

The transform methods are based on the analytic formulas based on the Radon Inversion Technique. The transform methods are of two types [5]:

(a) Direct Fourier Inversion (DFI)
(b) Convolution-Back-Projection (CBP).

In DFI method the direct Fourier transform of the projected data is taken and subsequent 2-d Fourier inversion leads to the reconstruction of the unknown distribution. In CBP method the data is convolved with a suitable filter

function and then back projection of the convolved data results in the reconstruction of the unknown distribution.

An important feature of the tomographic methods is that the point-density measurements can be made in a non-invasive manner without any prior knowledge of density distribution. In non-invasive methods the measurements are taken in such a way that the system does not get affected which is the case with tomographic methods.

The currently established reconstruction methods are mathematically and computationally complex, so the present work is an attempt to develop a simple algorithm to measure point densities in radially symmetric flow distributions. Such patterns are often encountered in gas-liquid flows through pipes. This chord-segment-inversion (CSI) algorithm has been demonstrated to be an extremely efficient method with further processing resulting in the radial density maps. The algorithm has been tested against some simulated radially symmetric distributions representing bubbly and annular flow distributions.

Additionally the CSI method has been applied to reconstruct density map for the air-water bubbly flow data [2,3,6]. The results appear to be comparable with the earlier known more complex tomographic methods [3,7,8]. The CPU time for the CSI technique is much less than that in other general tomographic methods.

CHAPTER 2

THEORETICAL FORMULATION

In this chapter a brief explanation of how the absorption coefficient is related to the source strength and detector reading along a particular chord is given. The geometry under consideration is the fan-beam geometry. The discrete form of this relationship has been explained and the chord-segment matrix and its triangularization has been introduced.

2.1 PRELIMINARIES

The single-beam radiation attenuation phenomenon is represented by

$$N = N_o \exp\left[- \int_c \mu(r,\emptyset)\, ds\right] \qquad (1)$$

where,

N = detector reading (count/second)

N_o = source strength (counts/second)

s = path of radiation (ray)

c = chord along which s is integrated

μ = absorption coefficient

$r,\emptyset$ = cylindrical coordinates.

Rewriting Eqn.(1), we have

$$d = \int_C \mu(r,\phi)\ ds \tag{2}$$

where,

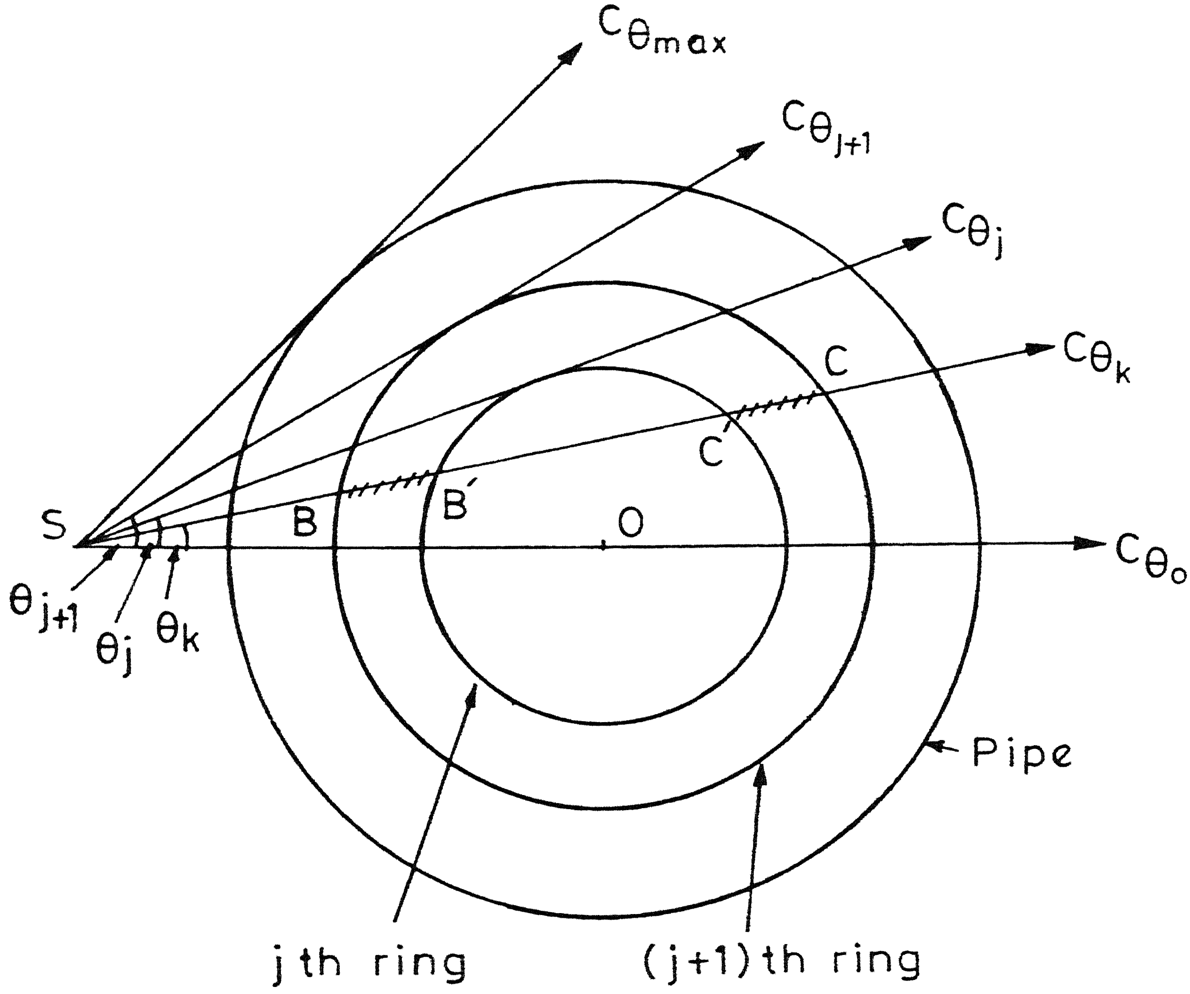
$$d = \ln (N_o/N) \tag{3}$$

In the fan-beam geometry (i.e. the geometry in which the beam diverges from the source), for a particular chord (ray), C_θ, corresponding to the ray making an angle θ from OS line (See Fig.1), the data is denoted by d_θ.

Thus,

$$d_\theta = \int_{C_\theta} \mu(r,\phi)\ ds \tag{4}$$

Here, d_θ, for different values of θ, is the data to be processed by the tomographic algorithms (in our case the tomographic algorithm is CSI). The reconstruction of the absorption coefficient, is done and the density, $\langle\rho\rangle$, (or void-fraction $\langle\alpha\rangle$) is determined by a calibration of the "CT numbers" (in this case μ) using some known density distributions. The reconstruction has been done for some known μ like 1, r, e^r, e^{-r} (See Fig.2). In other words, if a set of data d_θ for any density distribution is known to us then we can obtain the CT numbers for that distribution and hence by calibration the density.



$\rightarrow$ $S_{k,j} = BC - B'C'$

S - Source

O - Object centre

SO = D

Fig.1 Data collection geometry

The discrete form of Eqn.(2) can be written as (Fig.1),

$$d_k = \sum_{j=1}^{m} S_{k,j} \mu_j, \; k = 1,2,\ldots m \tag{5}$$

where,

$S_{k,j}$ = length of the segment of the kth ray falling in the jth ring (The hatched lines in Fig.1)

= BC – B'C'

μ_j = average value of μ in the jth ring μ

m = number of rings assumed within the object.

We note that radial symmetry is assumed and is now a function of r only. We also note that

$$S_{k,j} = 0, \text{ for } j < k \tag{6}$$

Since the kth ray does not intersect the jth ring if k is less than j. Eqn.(5) can be rewritten in matrix notation, as

$$[d] = [S][\mu] \tag{7}$$

where,

$[d] = (d_m \quad d_{m-1} \ldots\ldots d_1)$, the data vector,

$[\mu] = (\mu_m, \quad \mu_{m-1} \ldots\ldots \mu_1)$, the μ vector ,

and

$$[S] = \begin{bmatrix} S_{m,m} & 0 & 0 \\ S_{m-1,m} & S_{m-1,m-1} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ & & 0 \\ S_{1,m} & S_{2,m-1} & S_{1,1} \end{bmatrix}$$

is the chord-segment matrix which happens to be lower-triangular in this case.

By Eqn.(7), we get

$$[\mu] = [S]^{-1} [d] \tag{8}$$

Since the inverse of chord-segment matrix $[S]$ is involved, this method is known as chord-segment-inversion technique. This results in μ for various rings (or various intervals along the radius). A finer data vector will result in a better approximation of μ along the radial line.

The expression for $S_{k,j}$'s is given by (See Appendix-A)

$$S_{k,j} = BC - B'C' \quad \text{(See Fig. 1)}$$

$$= 2D\left[\sqrt{(\sin\theta_{j+1})^2-(\sin\theta_k)^2} - \sqrt{(\sin\theta_j)^2-(\sin\theta_k)^2}\right] \tag{9}$$

2.2 CHORD-SEGMENT-INVERSION (CSI) METHOD

The present chord-segment-inversion technique is relatively simple and faster than the general tomographic methods.

A FORTRAN program for the CSI algorithm has been written and implemented with the flexibility to change various geometrical variables, step size, error of integration, number of rings (See Appendix I) etc.

Now, for simulation studies, we want to obtain the data vector, [d], from Eqn.(4). Since we have assumed that $\mu(r,\emptyset)$ is radially symmetric function, i.e.

$$\mu(r,\emptyset) = \mu(r) \tag{10}$$

so Eqn.(4) can be rewritten as

$$d_\theta = \int_{C_\theta} \mu(r)\ ds\ . \tag{11}$$

For simplicity we replace the variable s by x and take the origin for x at the mid point of the chord. Thus,

$$d_\theta = \int_{C_\theta} \mu(r)\ dx \tag{12}$$

If for a chord at an angle θ, x_1 is the lower limit and x_2 is the upper limit for the variable x, then Eqn.(12) can be written as

$$d_\theta = \int_{x_1}^{x_2} \mu(r)\, dx \quad . \tag{13}$$

By the geometry (Fig.1) and Fig.A1 (Appendix-A)

$$r = \sqrt{(D \sin \theta)^2 + x^2} \tag{14}$$

Eqn.(13) reduces to

$$d_\theta = \int_{x_1}^{x_2} \mu(x)\, dx \tag{15}$$

The steps for the reconstruction of the density distribution are as follows:

(1) Read data [d] in form of a column vector for all rays in the fan-beam.

(2) Compute elements of the lower triangular [S] matrix using Eqn.(9) .

(3) Compute μ-values along the radial segments using Eqn.(8) .

(4) Calibrate μ-values to the density values.

For the Back substitution, averaging, plexi glass contribution and data see Appendices B,C,D and E .

We note that for simulation studies the data vector [d] will have to be generated by Eqn.(15).

CHAPTER-3

VALIDATION AGAINST SIMULATED DATA AND RESULTS FOR BUBBLY AIR-WATER FLOWS

3.1 VALIDATION AGAINST SIMULATED DATA

In this chapter, we discuss the results for the simulated data. We assume the radius of the pipe to be one unit and the distance of the source from the centre of the pipe to be two units. The pipe is further divided into twelve annular rings.

The CSI algorithm has been tested on the following assumed symmetric distributions:

$$\mu(r) = 1.0$$
$$\mu(r) = r$$
$$\mu(r) = \exp(r)$$
$$\mu(r) = \exp(-r) \quad .$$

For above mentioned test functions the errors in reconstruction are 10^{-6}, 0.0088, 0.03 and 0.005 respectively. Figure 2 shows the results of reconstruction along with the actual function distributions. For a listing of output see Appendix F.

The CSI algorithm has also been tested for annular flows (See Fig. 3), having the following two distributions:

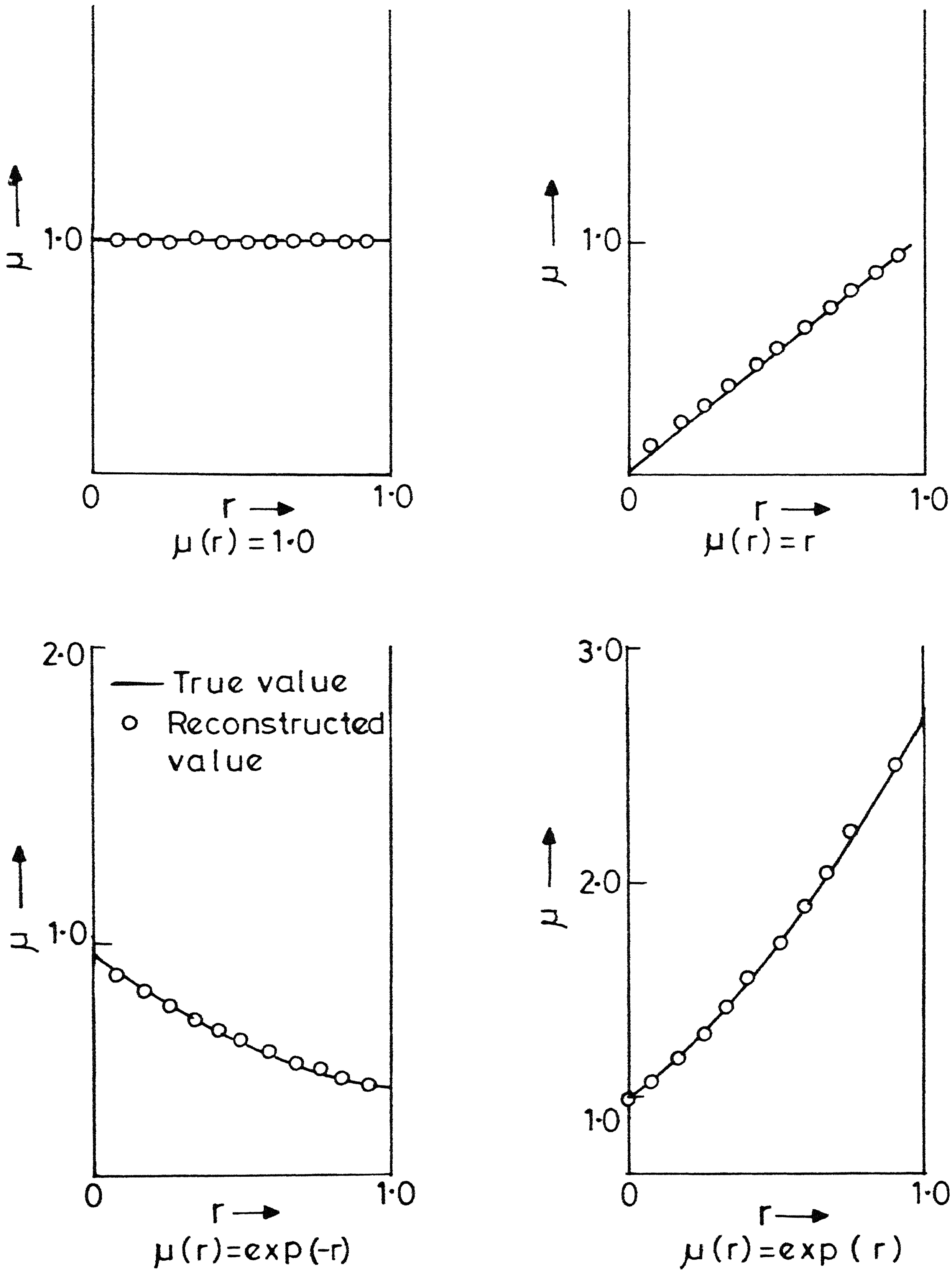


Fig.2 Reconstructed results for test-functions

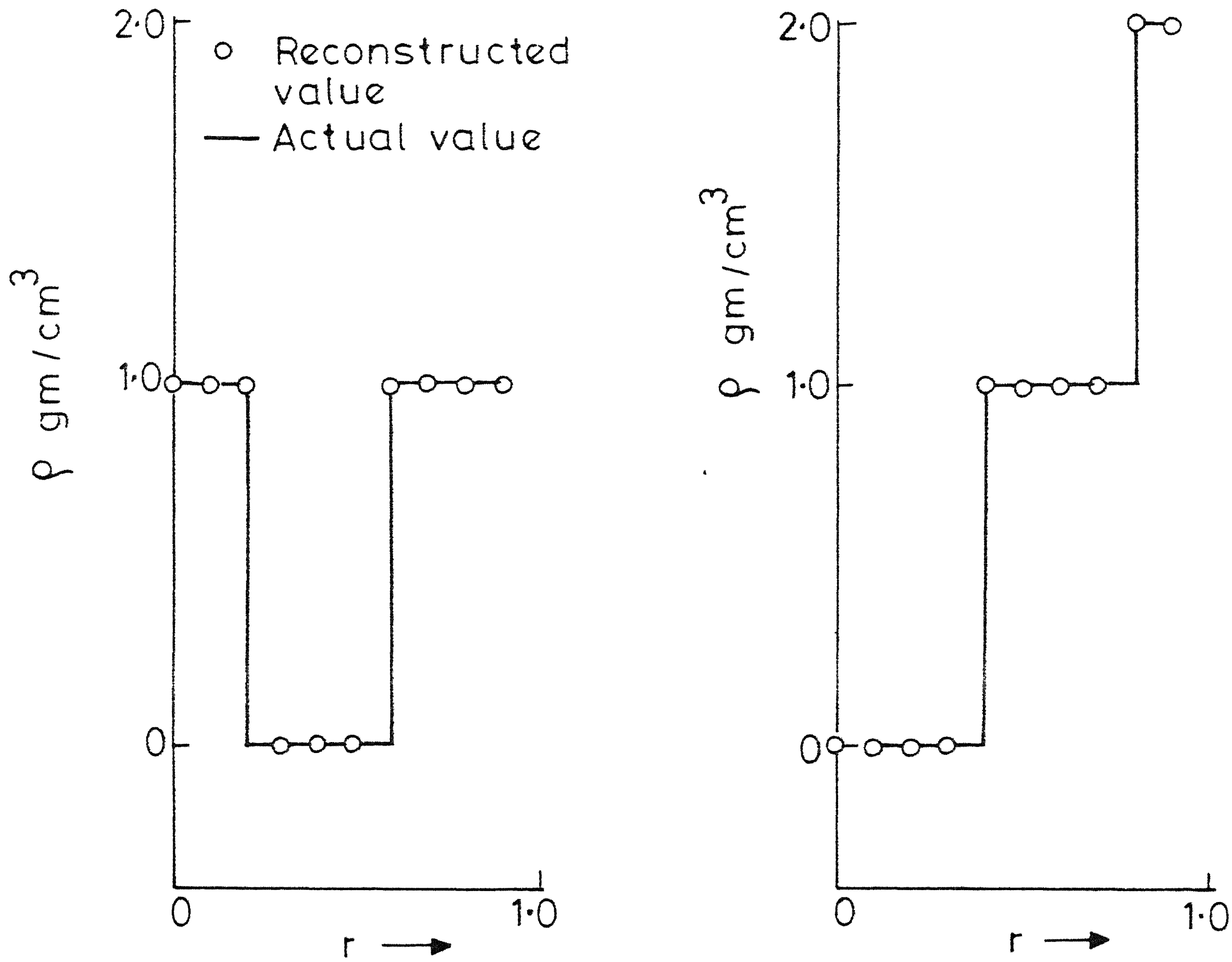


Fig. 3 Results for simulated annular flov

$$\mu(r) = \begin{cases} 1.0, & 0 \leq r \leq 0.2 \\ 0.0, & 0.2 < r \leq 0.6 \\ 1.0, & 0.6 < r \leq 1.0 \end{cases}$$

$$\mu(r) = \begin{cases} 0.0, & 0 \leq r \leq 0.4 \\ 1.0, & 0.4 < r \leq 0.8 \\ 2.0, & 0.8 < r \leq 1.0 \end{cases} .$$

The reconstruction errors are almost negligible (See Appendix G).

The above mentioned-distributions represent (in a calibrated sense) the various types of density/void fraction distribution encountered in radially symmetric-bubbly and annular flows.

The reconstruction μ-values matched the assumed-values very well for the simulated object of unit radius.

3.2 RESULTS FOR BUBBLY AIR-WATER FLOWS

Now here we will discuss the results for bubbly air-water flows. The data is taken from the study of Ref.[3,6]. Five different data-sets for four different cases of density (or void fraction) were processed by the CSI algorithm. The algorithm output, ⟨CTN⟩, had to be calibrated to obtain the density value. For this purpose, the previous work, included projection data for a few known cases of average density. Figure 4 (and Table F1, Appendix F)

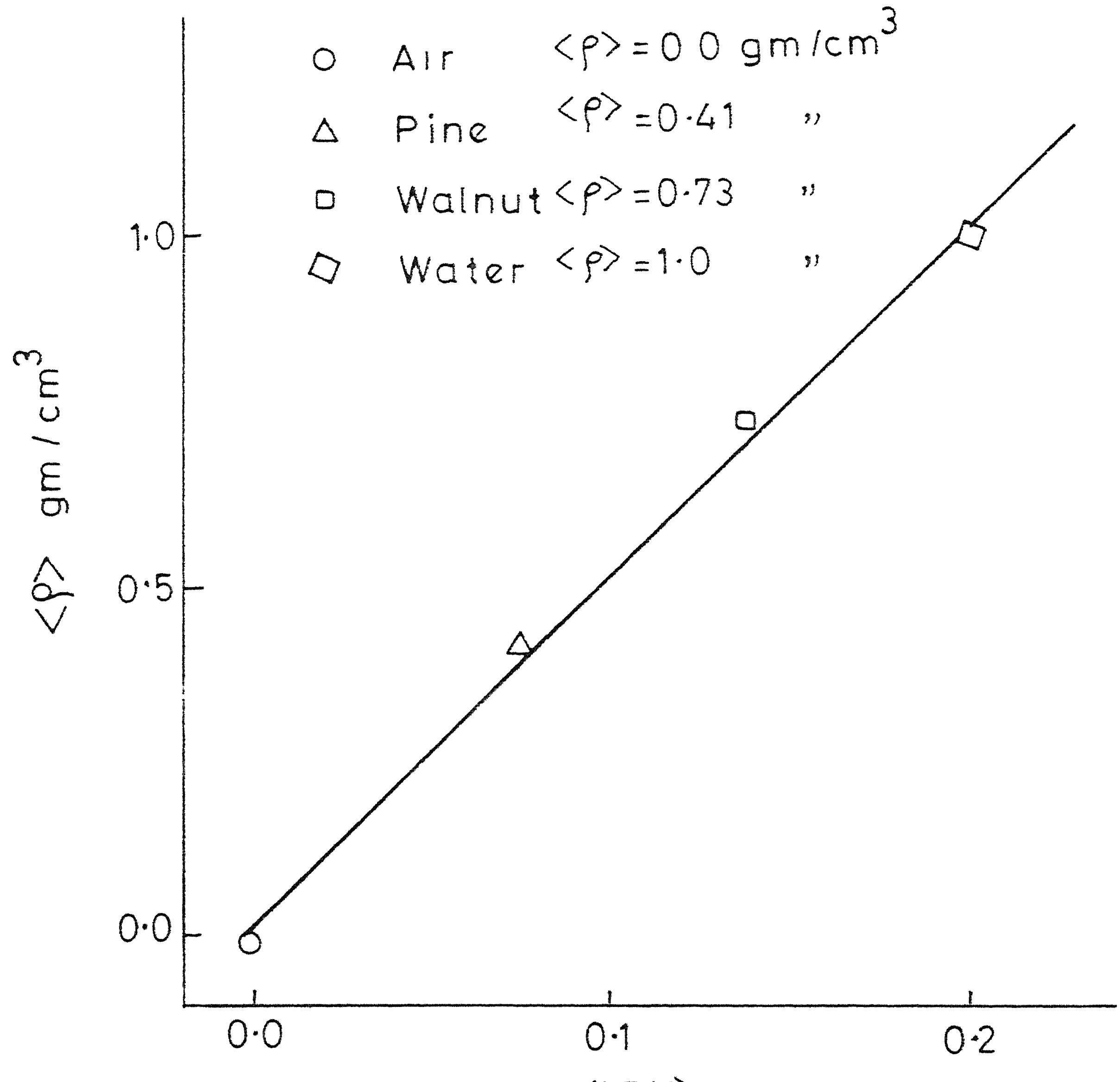


Fig. 4 Calibration curve

shows the calibration chart resulting from the processing of the projection data for four known cases of densities. The output, CTN, was corrected to the base of air, CTN = 0.0, to eliminate the effect of plexiglass. (Details in Appendix D).

The four points are joined by a straight line such that the line represents the best fitting line for these four points. This figure is now the calibration curve because now by knowing the $\langle CTN \rangle$ we can obtain the corresponding density value .

Similarly the data for all scans (See Appendix E) has been processed and we get the output (See Appendix H) in such a format that it gives the value of CTN for corresponding angle/radius. The value of ρ (obtained after calibration) for each radius has been plotted to show the reconstructed density and hence reconstructed profiles for various void fractions (See Figs 5-9).

In Figure 10 the comparison of reconstructed densities with actual densities has been shown by an alternative method [3.6] in which X-axis is the actual density and Y-axis is reconstructed density. A line such that $\langle \rho \rangle = \langle \rho_{CT} \rangle$ has been drawn and various densities obtained from calibration method are located to show the deviation of the reconstructed densities from the actual densities. Appendix F summarises the $\langle CTN \rangle$ and $\langle \rho \rangle$ results for all scans in a tabular form.

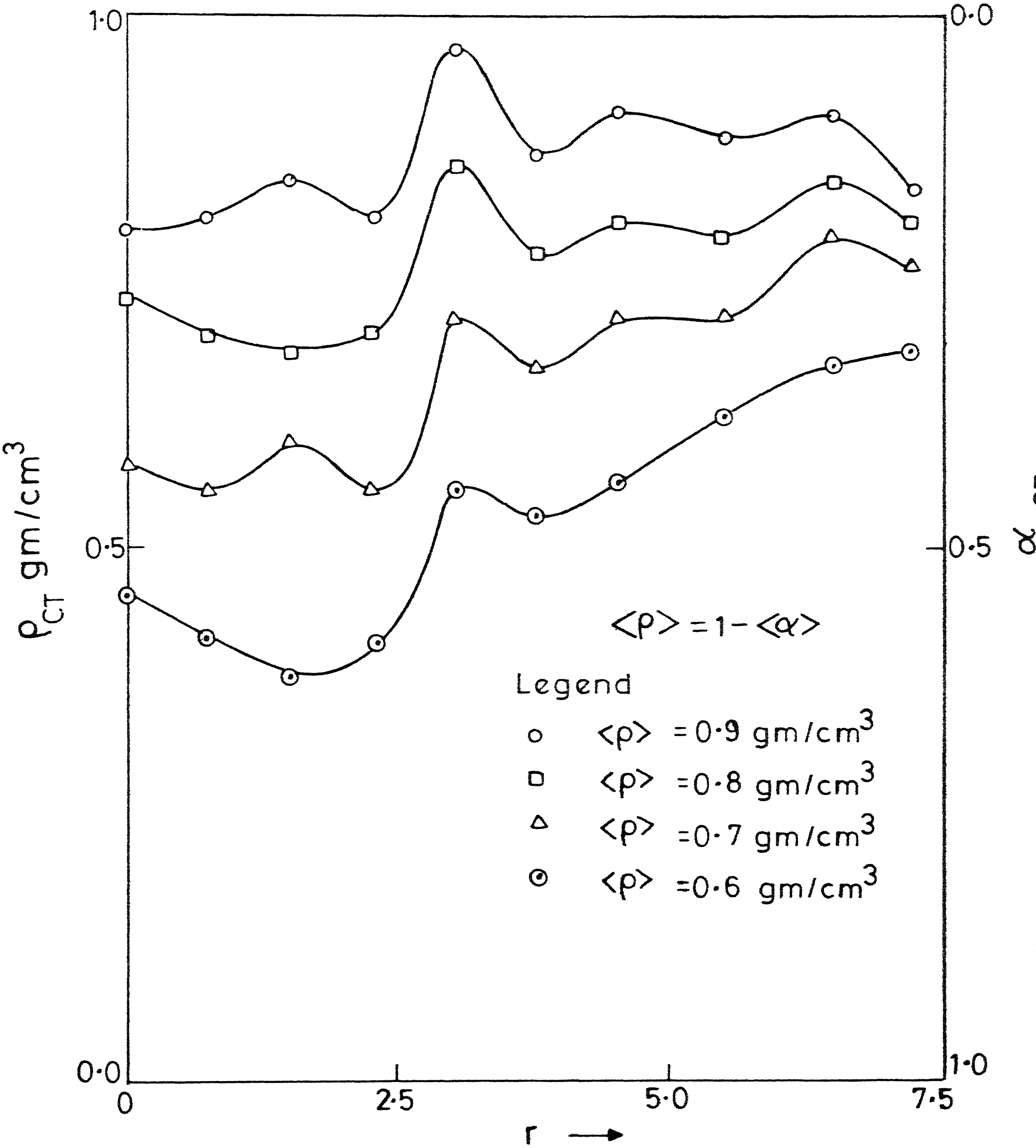


Fig.5 Reconstucted density profile for various density (average)cases for scan 1

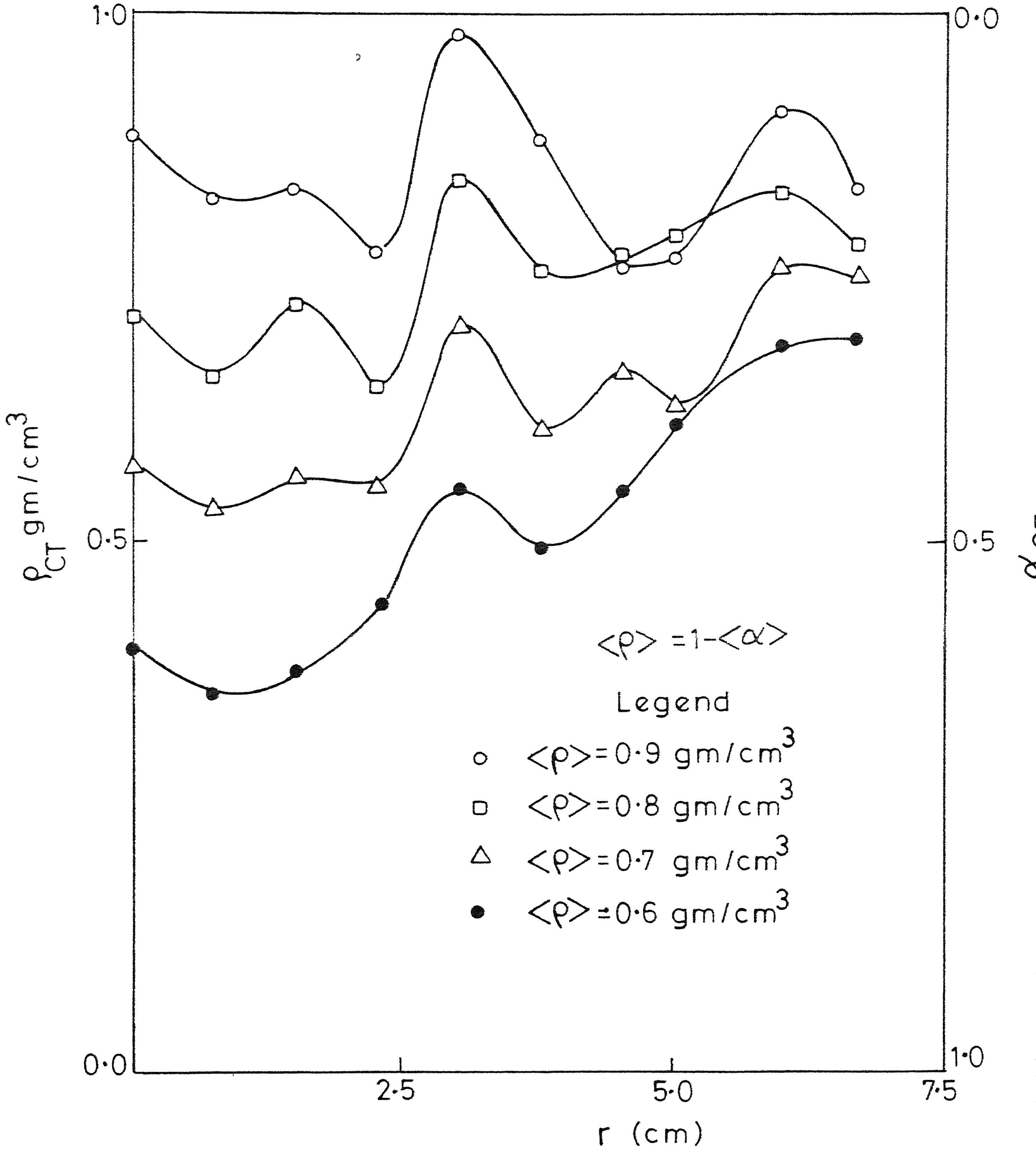


Fig.6 Density profile (radial) for scan 2

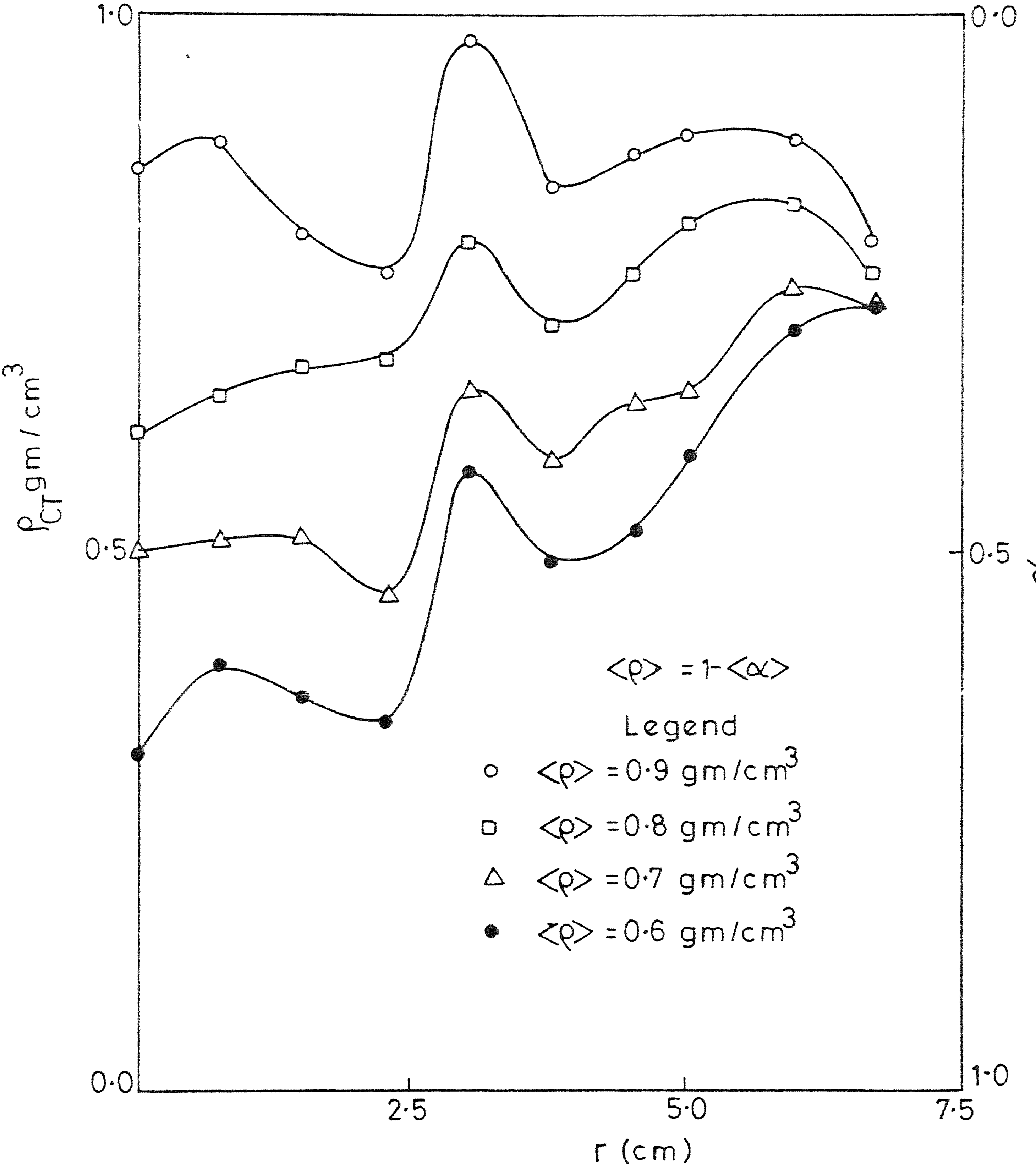


Fig.7 Density profile (radial) for scan 3

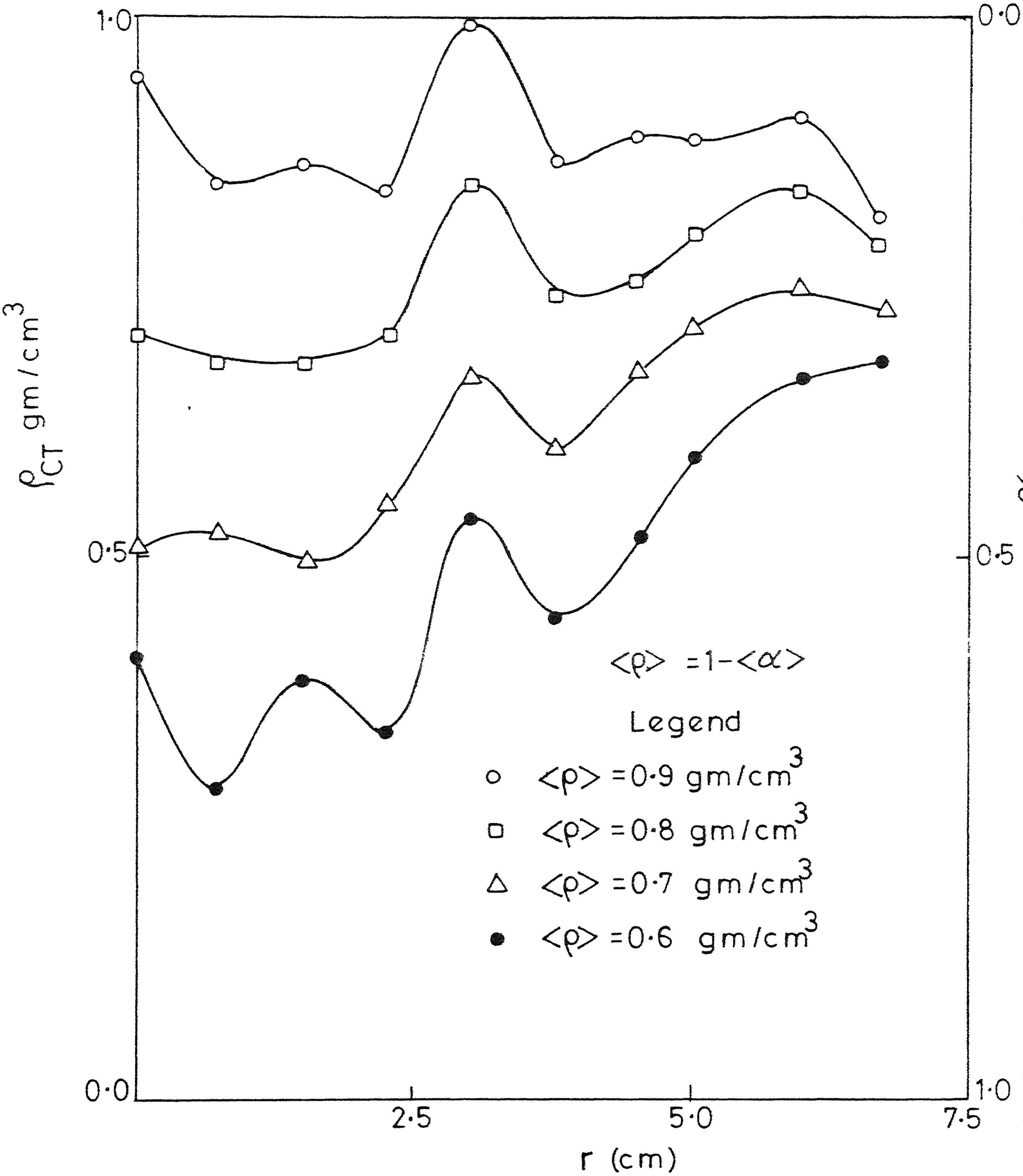


Fig.8 Density profile (radial) for scan 4

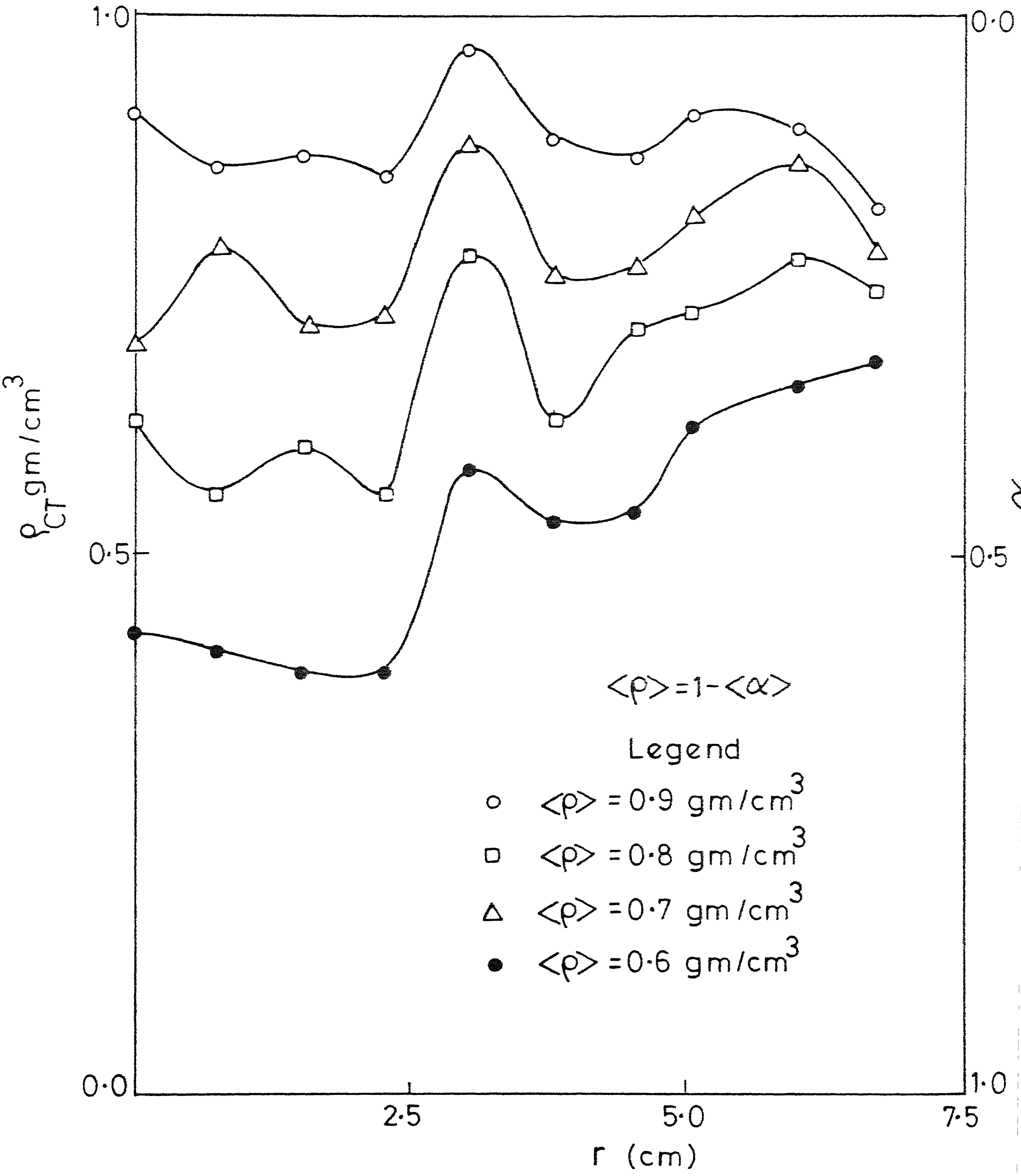


Fig.9 Density profile (radial) for scan 5

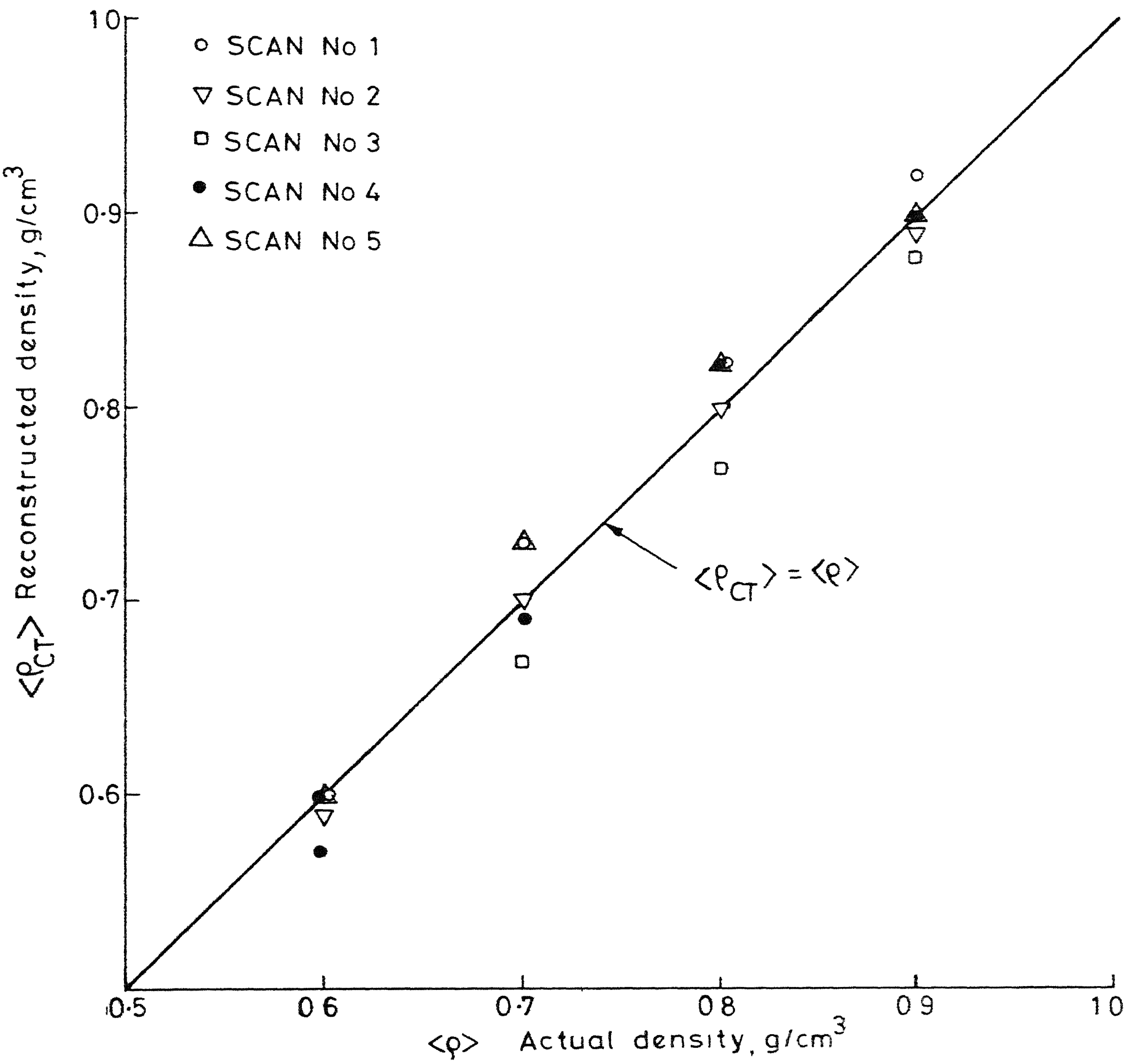


Fig.10 Comparison of reconstructed density with an alternate method

The maximum error obtained in reconstruction is $\pm$ 0.03 gm/cm^3. In Fig.11 the variation of error with density has been shown.

The maximum relative errors are + 4% and −5% respectively. In Fig.12 the variation of relative error with density has been shown.

We note that there are some statistical fluctuations in the count-rate recorded by the detector. This uncertainnity leads to an erroneous reconstruction. Such a discrepancy appears to be quite obvious for constant density cases of air, pine, walnut and water (See Fig. 13). However assuming Poisson distribution and applying $\pm 1\sigma$ and $\pm 3\sigma$ corrections (where 1σ implies one standard deviation of the count rate, N), the ripple appearing in Figure 13 is smoothned out as is evident from Figures 14 and 15. This exercise leads to a ρ_{CT} "band" for the air-water flow data. Since the point density values were not available to make any meaningful comparison, the air-water cases have not been presented.

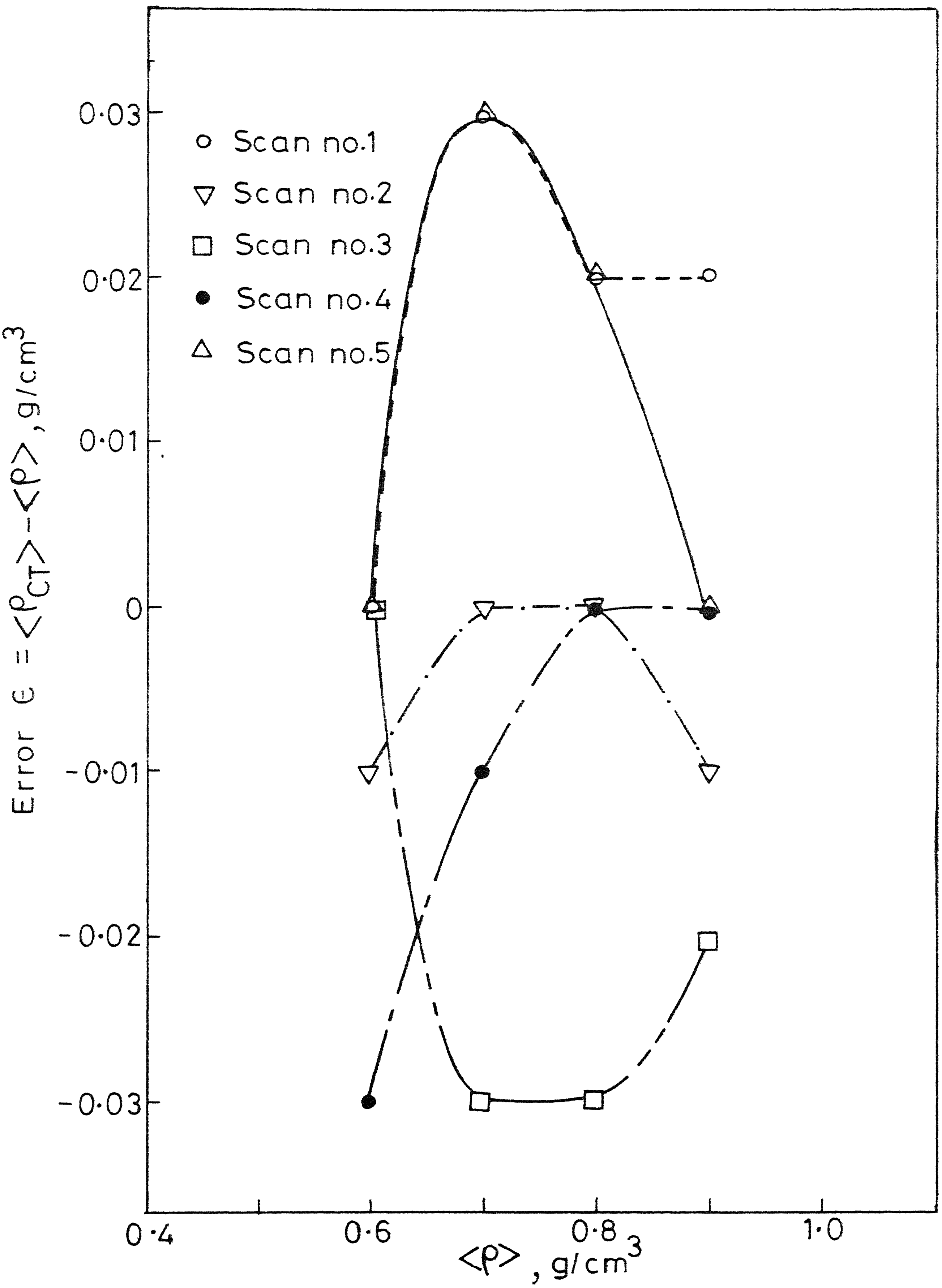


Fig.11 Error in density measurement for all scans

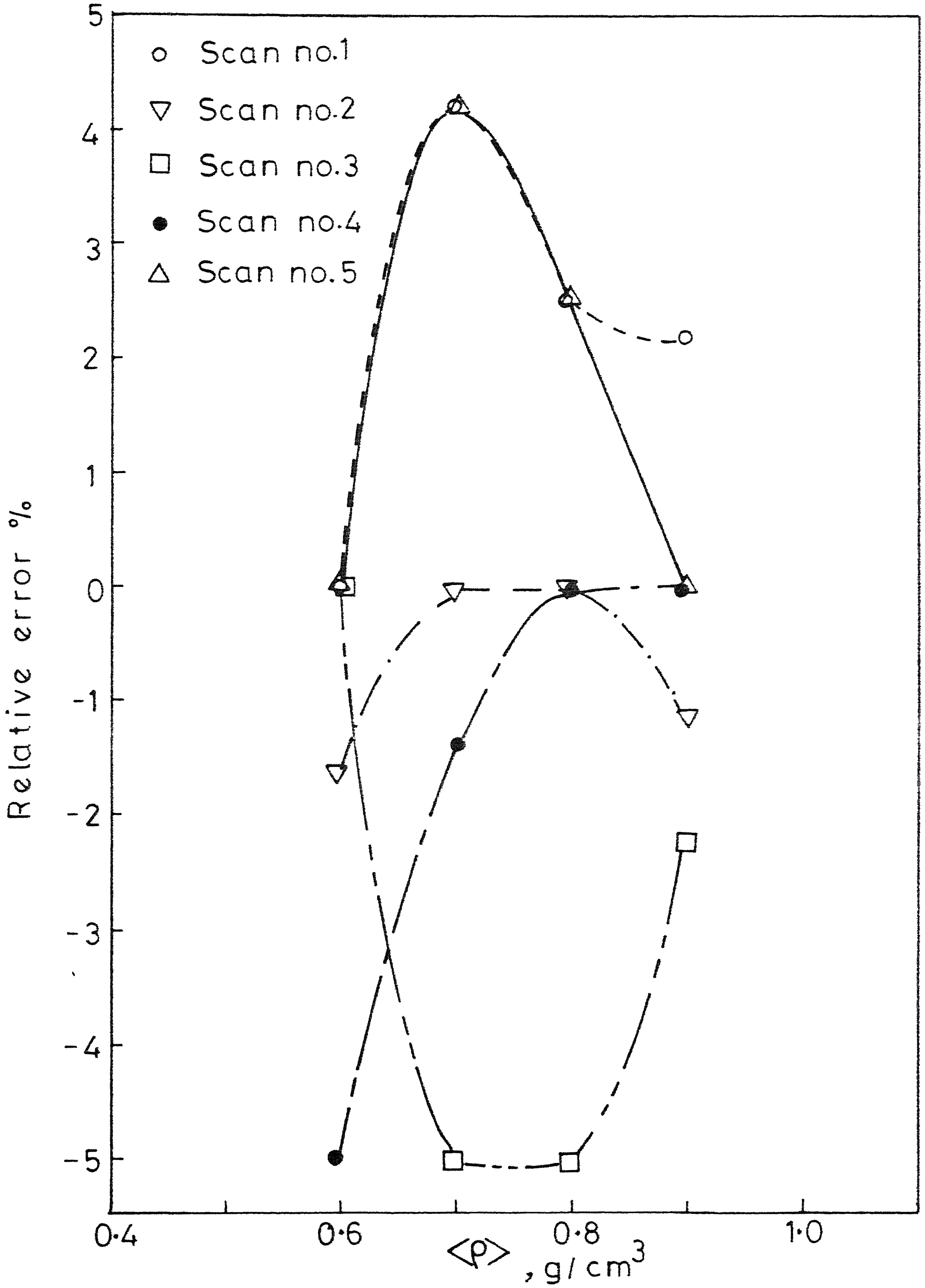


Fig.12 Relative error for all scans

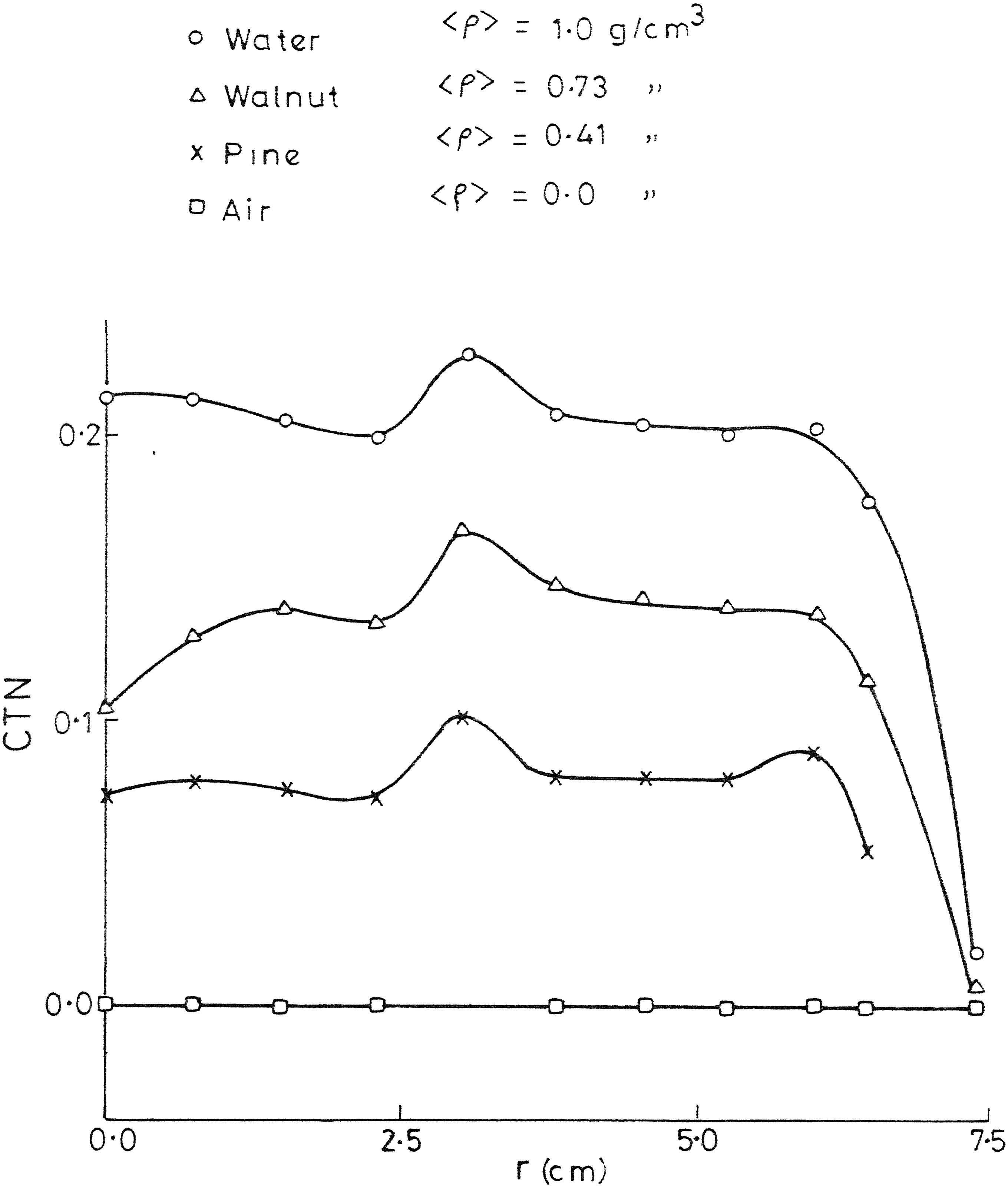


Fig.13 Calibration cases with mean value of counts

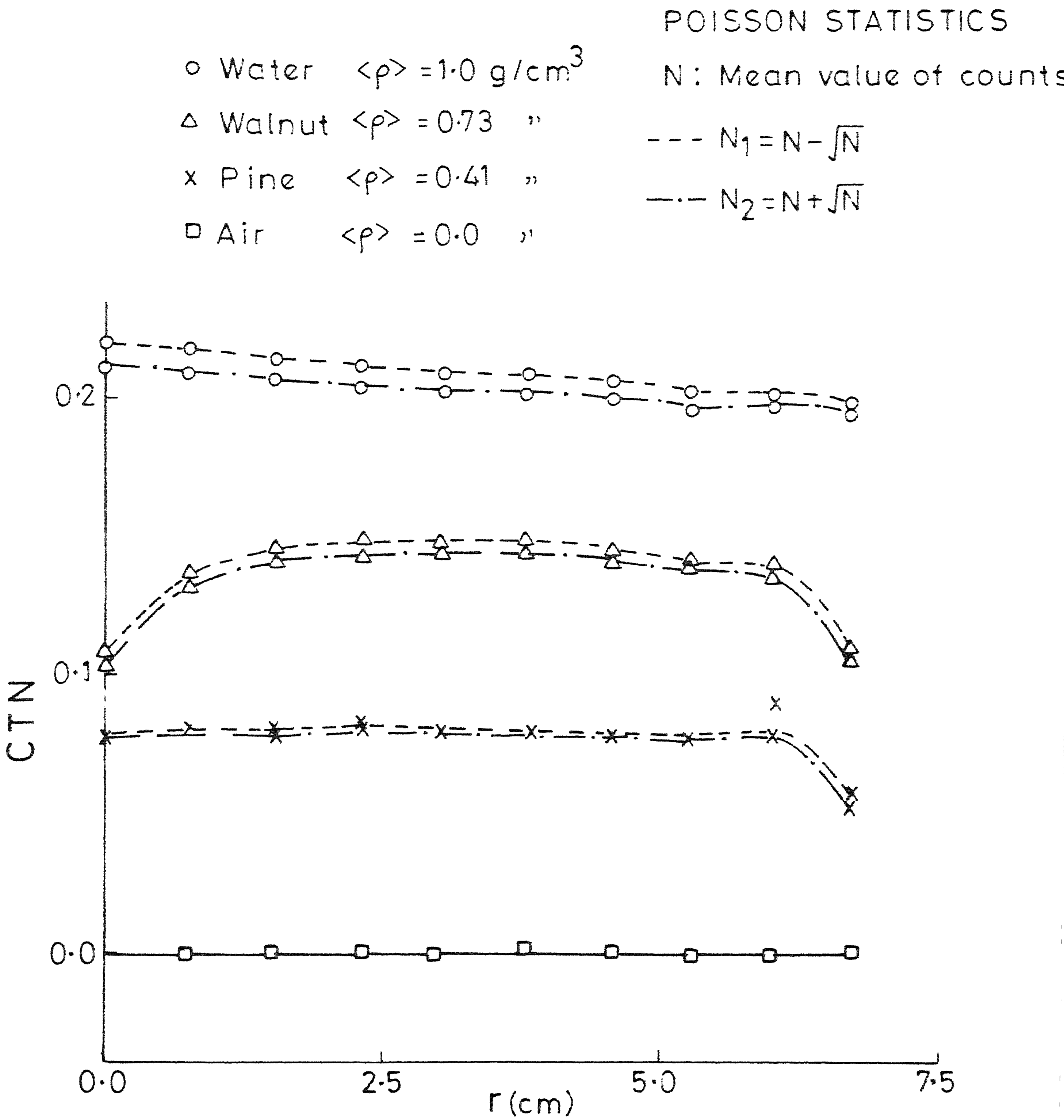


Fig.14 Calibration cases with 1-σ band

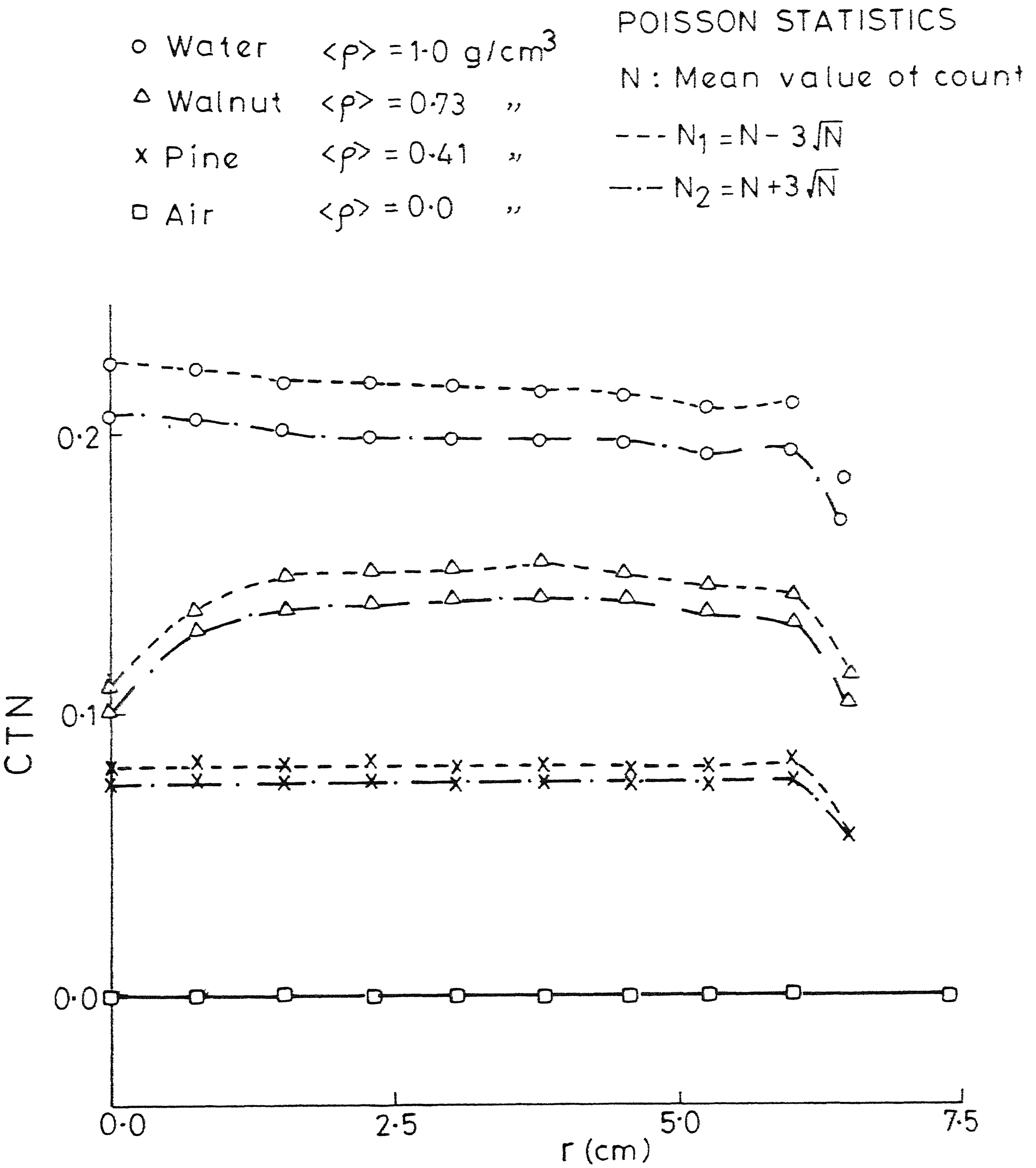


Fig.15 Calibration cases with 3-σ band

CHAPTER-4

DISCUSSION

The proposed chord-segment inversion (CSI) technique has been demonstrated to be applicable to radially symmetric two-phase flows. The results are comparable to the results obtained by more general methods of tomographic reconstruction [2,3,6,7,8].

However, taking the advantage of the radial symmetry in the theoretical formulation itself, the CSI method consumes less Central Processing Unit (CPU) time and it is simple to use.

Comparable results have been obtained by CSI technique for density measurement in bubbly air-water flow for cases $0.6 \leq \rho \leq 1.0$. The maximum deviation of ρ_{CT} is about $\pm$ 0.03g/cm^3 which is approximately $\pm$5%. The maximum relative errors are +4% and -5% respectively. The error in the point density values could not be estimated as no alternate method was available in the study of Ref.[3,6]. But the test-function results indicate that for "pure" data the reconstruction is exact. The results of CSI method are also comparable to "radial polynomial" method already suggested and tested [9] for radially symmetric flow distributions.

APPENDIX-A

Suppose we have a pipe of radius R and a gamma ray source S at a distance D from the centre of the pipe. Consider now jth and (j+1)th annular rings and five chords C_{θ_o}, C_{θ_k}, C_{θ_j}, $C_{\theta_{j+1}}$ and $C_{\theta_{max}}$ such that they make angles 0, θ_k, θ_j, θ_{j+1}, θ_{max} as shown in Fig.1.

The angle corresponding to chord $C_{\theta_{max}}$ is the maximum angle θ_{max} and is given by

$$\sin(\theta_{max}) = R/D \tag{A-1}$$

$$\theta_{max} = \sin^{-1}(R/D) \tag{A-2}$$

Note that any ray (chord) C_{θ_n} is the tangent to the $(n-1)^{th}$ annular ring.

Now we want to calculate the length of the segment of the kth ray falling in the jth annular ring that is $S_{k,j}$.

From Fig. 1 this length is the hatched line

$$S_{k,j} = BC - B'C' \tag{A-3}$$

Now see the detailed geometry of Fig. 1 in Fig.A1.

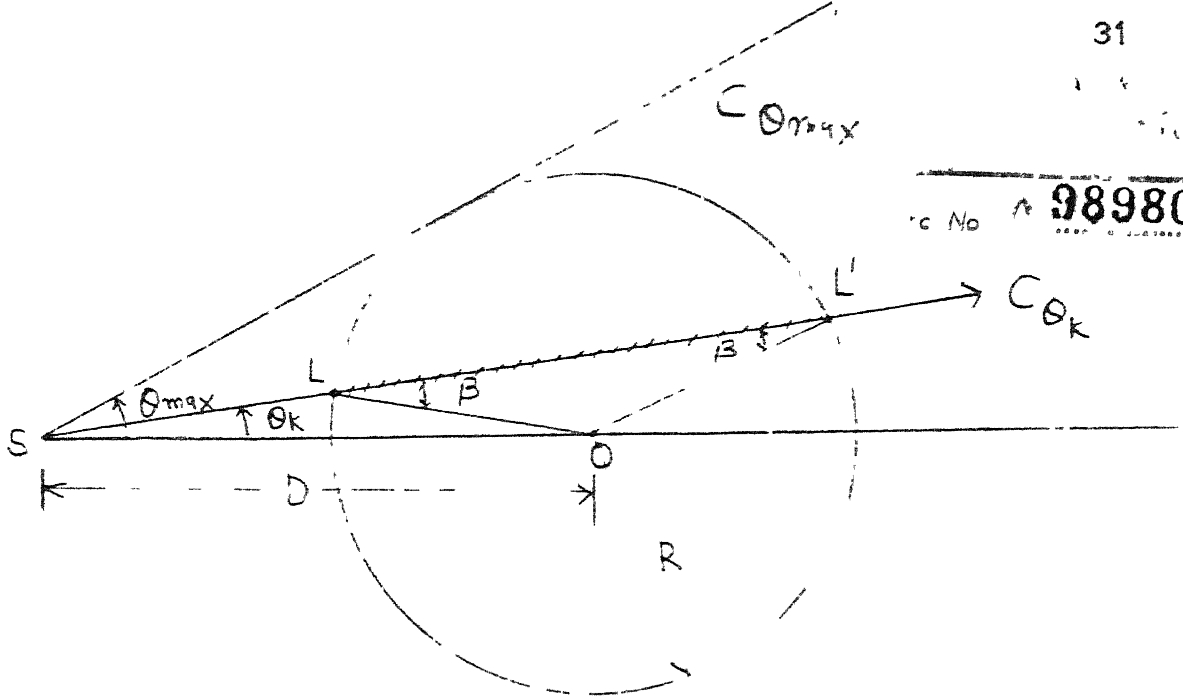


Figure A1 : Fan Beam Geometry Details

We are interested in calculating the length LL' in the C_{θ_k} chord such that $C_{\theta_{max}}$ is the tangential chord.

From the triangle SOL'

$$\frac{D}{\mathrm{Sin}(\beta)} = \frac{R}{\mathrm{Sin}(\theta_k)} \tag{A-4}$$

$$\beta = \mathrm{Sin}^{-1}\left(\frac{D\ \mathrm{Sin}(\theta_k)}{R}\right) \tag{A-5}$$

But from Eqn.(A-1)

$$R = D\ \mathrm{Sin}\ (\theta_{max}) \tag{A-6}$$

So from Eqn.(A-5)

$$\beta = \mathrm{Sin}^{-1}\left(\frac{D\ \mathrm{Sin}(\theta_k)}{D\ \mathrm{Sin}(\theta_{max})}\right) \tag{A-7}$$

$$= \sin^{-1}\left(\frac{\sin(\theta_k)}{\sin(\theta_{max})}\right) \tag{A-8}$$

The length LL'

$$LL' = 2R\cos(\beta) \tag{A-9}$$

$$= 2D\sin(\theta_{max})\cos\left(\sin^{-1}\left(\frac{\sin(\theta_k)}{\sin(\theta_{max})}\right)\right) \tag{A-10}$$

$$= 2D\sin(\theta_{max})\sqrt{1 - \frac{\sin^2\theta_k}{\sin^2\theta_{max}}} \tag{A-11}$$

$$= 2D\sqrt{\sin^2\theta_{max} - \sin^2\theta_k} \tag{A-12}$$

By this token we have in Fig. 1.

$$BC = 2D\sqrt{\sin^2\theta_{j+1} - \sin^2\theta_k} \tag{A-13}$$

$$B'C' = 2D\sqrt{\sin^2\theta_j - \sin^2\theta_k} \tag{A-14}$$

hence the length

$$S_{k,j} = BC - B'C'$$

$$= 2D\left[\sqrt{\sin^2\theta_{j+1} - \sin^2\theta_k} - \sqrt{\sin^2\theta_j - \sin^2\theta_k}\right]$$

(A-15)

APPENDIX-B

The Back Substitution is as follows:

Recalling equation (7)

$$[d] = [S]\,[\mu] \tag{B-1}$$

The expanded form of this equation is

$$\begin{Bmatrix} d_m \\ d_{m-1} \\ \vdots \\ d_1 \end{Bmatrix} = \begin{bmatrix} S_{m,m} & & 0 \\ S_{m-1,m} & S_{m-1,m-1} & \vdots \\ \vdots & \vdots & 0 \\ S_{1,m} & S_{2,m-1} & S_{1,1} \end{bmatrix} \begin{bmatrix} \mu_m \\ \mu_{m-1} \\ \vdots \\ \mu_1 \end{bmatrix} \tag{B-2}$$

Then the algebraic equations will be

$$d_m = S_{m,m}\,\mu_m \tag{B-3}$$

$$d_{m-1} = S_{m-1,m}\,\mu_m + S_{m-1,m-1}\,\mu_{m-1} \tag{B-4}$$

$$\begin{aligned} d_{m-2} &= S_{m-2,m}\mu_m + S_{m-2,m-1}\,\mu_{m-1} + S_{m-2,m-2}\,\mu_{m-2} \\ &\vdots \end{aligned} \tag{B-5}$$

$$d_1 = S_{1,m}\mu_m + S_{2,m-1}\mu_{m-1} + \dots + S_{1,1}\mu_1 \tag{B-6}$$

From Equation (B-3) we have,

$$\mu_m = \frac{d_m}{S_{m,m}}$$

Substituting this value of μ_m in (B-4) we obtain the value of μ_{m-1} and then using the value of μ_{m-1} in next equation to get μ_{m-2} and so on.

So by back substitution we get the values of all μ's.

<u>APPENDIX C</u>

For the simulated data study of radially symmetric distributions, the μ has been reconstructed. In Fig. C1, μ_{j-1}, μ_j and μ_{j+1} are the reconstructed values of μ's corresponding to radius O, R_j and R_{j+1} respectively. Similarly $\bar{\mu}_{j-1}$, $\bar{\mu}_j$ and $\bar{\mu}_{j+1}$ are the actual μ's at radius O, R_j and R_{j+1} respectively.

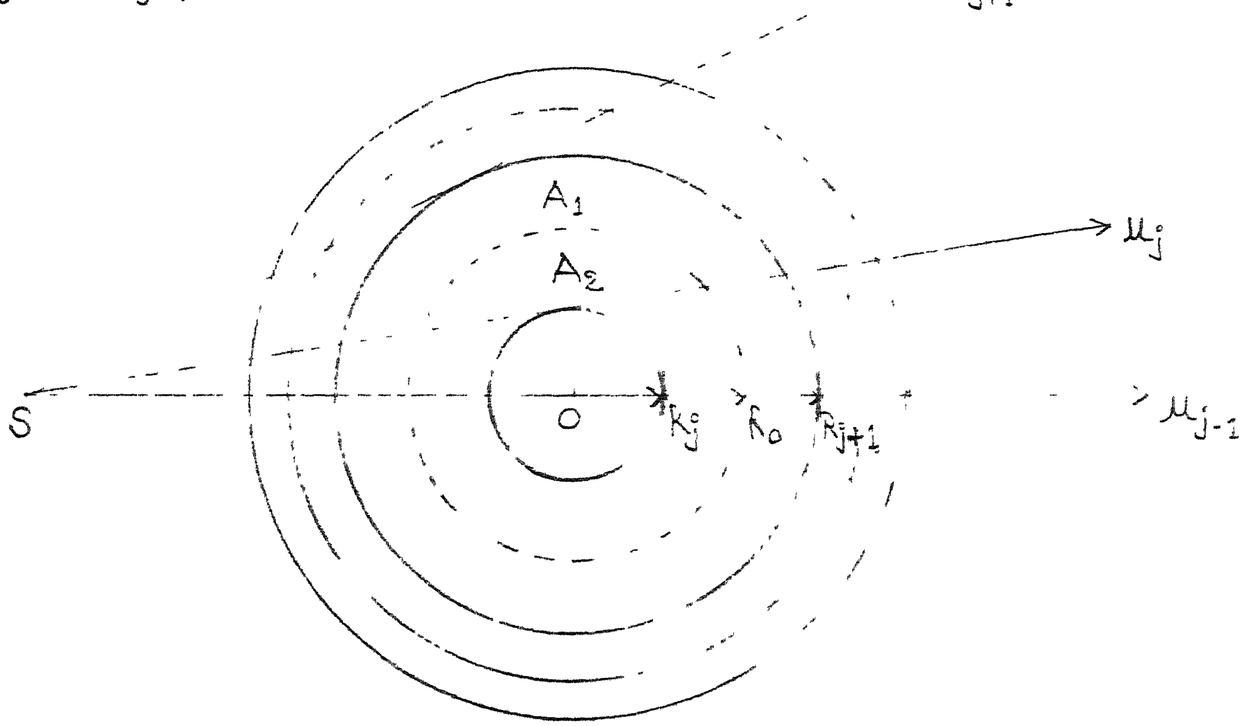


Fig. C1 AVERAGE-VALUES

Now take the average values of $\bar{\mu}$'s $\langle\bar{\mu}_{j-1}\rangle$, $\langle\bar{\mu}_j\rangle$ and $\langle\bar{\mu}_{j+1}\rangle$ such that they are the values of $\bar{\mu}$'s in first, second and third annular rings. Now $\langle\bar{\mu}_j\rangle$ is the value of $\bar{\mu}$ at radius R_o as shown in Fig. C1.

If area between R_{j+1} and R_o is A_1 tnen

$$A_1 = \pi R_{j+1}^2 - \pi R_o^2$$

Similarly if area between R_o and R_j is A_2 then

$$A_2 = \pi R_o^2 - \pi R_j^2$$

Taking $A_1 = A_2$

$$R_o = \sqrt{\frac{R_j^2 + R_{j+1}^2}{2}}$$

i.e. the average radius of the annular ring having R_j internal radius and R_{j+1} outer radius. This must be taken into account when calculating μ at various radii.

APPENDIX D

The data for all scans has been taken from [6]. There was plexi-glass around the pipe in the experiment of [6] for all scans. For air the CTN (absorption coefficient) must be zero but for air data set we do not get the CTNs (corresponding to different angles) equal to zero but some non-zero values, this is due to the plexi-glass whatever CTNs we get will be the CTNs for plexi-glass but not for air.

So this plexi-glass contribution must be taken into account in order to obtain the correct values of CTNs for all cases. For this we make air as the reference and all CTNs equal to zero by subtracting CTNs of plexi-glass. Similarly for any case we subtract the CTNs of plexi-glass to get the correct CTNs and hence correct <CTNs>.

APPENDIX F

TABLE-F1

(Results for air-pine walnut and water)

Case	$\langle \rho \rangle$, g/cm^3	$\langle CTN \rangle$
Air	0.0	0.000
Pine	0.41	0.075
Walnut	0.732	0.137
Water	1.00	0.201

TABLE-F2

(Reconstructed Densities)

Scan No. \ $\langle \rho \rangle$ gm/cm^3	0.6	0.7	0.8	0.9
1	0.60	0.73	0.82	0.92
2	0.59	0.70	0.8	0.89
3	0.60	0.67	0.77	0.88
4	0.57	0.69	0.80	0.90
5	0.6	0.73	0.82	0.90

TABLE-F3

(Reconstructed <CTNs>)

Scan No. \ $<\rho>$ gm/cm^3	0.6	0.7	0.8	0.9
1	0.116	0.141	0.159	0.175
2	0.114	0.136	0.155	0.173
3	0.111	0.129	0.152	0.171
4	0.110	0.134	0.155	0.176
5	0.115	0.141	0.159	0.176

Table F 1 Scan No. 1

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR	2869	2063	2084	2315	2404	2447	2485	2490	2516	2528	2534	2532	2527	2522	2526	2513	2492	2471	2449	2398	2308	2149	1743	2834	2842
46% VOID	2875	2109	1573	1518	1483	1448	1435	1434	1443	1456	1465	1464	1470	1469	1463	1466	1456	1423	1452	1464	1482	1541	1703	2830	2832
40% VOID	2826	2100	1552	1469	1402	1380	1351	1339	1332	1345	1349	1348	1342	1348	1361	1345	1357	1342	1364	1384	1422	1497	1667	2835	2846
30% VOID	2857	2061	1517	1377	1278	1218	1176	1160	1146	1130	1133	1126	1136	1139	1141	1158	1171	1176	1209	1269	1314	1430	1646	2835	2837
20% VOID	2864	2074	1500	1331	1218	1133	1079	1055	1018	1010	1002	1001	999	1005	1018	1023	1048	1077	1125	1198	1261	1390	1630	2824	2819
10% VOID	2879	2035	1505	1308	1173	1082	1005	958	913	894	884	876	894	894	902	923	952	991	1041	1127	1218	1370	1632	2800	2815
WATER	2885	2062	1495	1266	1106	996	913	858	811	781	758	748	740	745	765	790	826	877	951	1044	1153	1359	1663	2850	2835
WALNUT (ρ=.732)	2853	2082	1742	1565	1449	1392	1335	1265	1206	1159	1141	1121	1126	1108	1113	1132	1161	1207	1282	1365	1458	1619	1705	2846	2846
PINE (ρ=.41)	2858	2060	1773	1863	1800	1754	1705	1678	1639	1616	1601	1599	1595	1594	1610	1621	1645	1669	1715	1750	1782	1856	1710	2834	2827

Table E.2   Scan No. 2

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2574												2878
40% VOID	2901	2124	1563	1485	1429	1378	1359	1356	1358	1350	1374	1380	1387	1392	1389	1379	1386	1380	1387	1403	1435	1525	1749	2878	2883
30% VOID	2914	2103	1548	1411	1322	1268	1218	1203	1186	1183	1184	1181	1188	1193	1196	1203	1225	1246	1272	1322	1359	1476	1737	2862	2879
20% VOID	2907	2089	1541	1379	1266	1188	1126	1097	1062	1018	1043	1038	1040	1048	1047	1077	1090	1123	1176	1236	1310	1453	1735	2882	2874
10% VOID	2902	2090	1532	1342	1198	1108	1037	985	950	922	907	908	910	919	932	959	975	1020	1093	1166	1254	1435	1724	2883	2885
WATER													746												

Table I.3 Scan No. 3

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2605												2917
40% VOID	2944	2156	1595	1509	1440	1404	1349	1370	1386	1381	1399	1401	1426	1411	1424	1421	1400	1411	1432	1442	1472	1550	1767	2936	2938
30% VOID	2965	2172	1589	1447	1379	1308	1283	1257	1247	1236	1254	1256	1251	1249	126[illegible]	1276	1277	1285	1308	1358	1386	1504	1750	2910	2912
20% VOID	2349	2119	1550	1380	1253	1185	1136	1097	1072	1054	1050	1063	1077	1073	1079	1088	1128	1152	1188	1241	1330	1480	1759	2920	2917
10% VOID	2950	2120	1542	1342	1203	1110	1038	991	948	942	926	918	922	924	955	973	990	1043	1096	1170	1279	1454	1743	2917	2926
WATER													763												

Table E.4 Scan No. 4

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2575												2893
40% VOID	2895	2124	1574	1501	1443	1426	1405	1396	1406	1424	1413	1421	1422	1438	1414	1422	1413	1418	1410	1416	1446	1524	1728	2868	2860
30% VOID	2890	2095	1543	1416	1312	1278	1224	1206	1203	1195	1197	1200	1210	1208	1221	1209	1239	1248	1264	1307	1372	1486	1735	2857	2860
20% VOID	2884	2080	1530	1351	1234	1170	1105	1061	1041	1040	1026	1030	1034	1038	1049	1053	1087	1122	1171	1225	1303	1456	1722	2856	2853
10% VOID	2885	2075	1518	1311	1181	1083	1007	964	926	905	891	889	882	895	906	927	962	1014	1070	1151	1246	1421	1726	2860	2862
WATER													759												

Table E.6 Scan No.5,

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2575												2900
40% VOID	2897	2123	1566	1499	1428	1391	1355	1352	1340	1342	1361	1364	1367	1369	1379	1372	1370	1369	1402	1411	1460	1531	1739	2895	2885
30% VOID	2916	2121	1559	1413	1316	1237	1194	1174	1164	1145	1141	1143	1144	1153	1154	1169	1174	1218	1234	1290	1352	1474	1729	2866	2875
20% VOID	2909	2091	1534	1356	1227	1159	1090	1052	1026	1010	997	1001	1006	999	1027	1035	1065	1108	1159	1212	1287	1452	1729	2874	2882
10% VOID	2906	2102	1521	1308	1172	1079	1009	967	918	907	881	887	886	894	908	930	972	1011	1080	1146	1258	1423	1729	2884	2879
WATER													757												

```
RADIUS  R=  1.00000000
DISTANCE OF SOURCE  D=  2.00000000
STARTING  ANGLE  TH=  0.00000000
STEP OF ANGLE  DTH=  2.50000000
ERROR OF INTEGRATION  E=  0.00000010
NUMBER OF ANNULUS RINGS  N=  12
```

TABLE:DISTRIBUTION-F(r)=1.000

TH	DATA	r	F(r)	FZ(r)	PRODUCT
0.00	2.00000000	0.05168713	1.00000000	1.00000010	0.02390942
2.50	1.99237480	0.13783159	1.00000000	0.99999987	0.07154528
5.00	1.96938110	0.22196040	1.00000000	0.99999997	0.11853865
7.50	1.93064930	0.30721581	1.00000000	1.00000000	0.15482811
10.00	1.87551090	0.39242782	1.00000000	1.00000000	0.20975314
12.50	1.80290380	0.47714443	1.00000000	0.99999999	0.25310179
15.00	1.71119940	0.56109051	1.00000000	1.00000000	0.29451405
17.50	1.59787870	0.64405242	1.00000000	0.99999999	0.33358492
20.00	1.45888850	0.72584350	1.00000000	0.99999999	0.37031539
22.50	1.28718850	0.80629127	1.00000000	0.99999999	0.40412931
25.00	1.06878470	0.88523215	1.00000000	0.99999999	0.43485569
27.50	0.76721019	0.95250900	1.00000000	1.00000000	0.45229441

AVERAGE DISTRIBUTION =1.00000000

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=r

TH	DATA	r	F(r)	F2(r)	PRODUCT
0.00	1.00000000	0.05168713	0.05168713	0.06833501	0.00163385
2.50	1.02001120	0.13783159	0.13783159	0.13832951	0.00989597
5.00	1.05859700	0.22196040	0.22195040	0.22091244	0.02620875
7.50	1.10289510	0.30721581	0.30721581	0.30511155	0.05024095
10.00	1.14510510	0.39242782	0.39242782	0.38943954	0.03170054
12.50	1.17876480	0.47714443	0.47714443	0.47348329	0.11983943
15.00	1.19768690	0.56109051	0.56109051	0.55574642	0.15395065
17.50	1.19523980	0.64405242	0.64405242	0.63899912	0.21322433
20.00	1.16344980	0.72584350	0.72584350	0.71995514	0.25651482
22.50	1.09130230	0.80629127	0.80529127	0.79930034	0.32402073
25.00	0.96038473	0.88523215	0.88523215	0.87539731	0.33111501
27.50	0.72839410	0.96250900	0.96250900	0.94940629	0.43890513

AVERAGE DISTRIBUTION = .65094534

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=EXP(-r)

TH	DATA	r	F(r)	FC(r)	PRODUCT
0.00	1.26424110	0.06168713	0.94017599	0.93358303	0.02244093
2.50	1.23982890	0.13783159	0.87124540	0.87427597	0.05255120
5.00	1.18751500	0.22196040	0.80094708	0.80444957	0.09543881
7.50	1.11873130	0.30721581	0.73549186	0.73913804	0.12183073
10.00	1.03903740	0.39242782	0.67541510	0.67905950	0.14244375
12.50	0.95179631	0.47714443	0.62055289	0.62415413	0.15797447
15.00	0.85905208	0.55109051	0.57058649	0.57411547	0.15908503
17.50	0.76183482	0.64405242	0.52515993	0.52862950	0.17639574
20.00	0.66017219	0.72584350	0.48391622	0.48737543	0.18048303
22.50	0.55274329	0.80629127	0.44551099	0.45005970	0.18188533
25.00	0.43562403	0.88523215	0.41251837	0.41654013	0.18113945
27.50	0.29696561	0.95250900	0.38193341	0.38707200	0.17894124

AVERAGE DISTRIBUTION = .63177192

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=EXP(r)

TH	DATA	r	F(r)	FC(r)	PRODUCT
0.00	3.43656360	0.05168713	1.05362950	1.07844110	0.02578490
2.50	3.45351180	0.13783159	1.14778220	1.15483840	0.03262793
5.00	3.48265620	0.22196040	1.24852190	1.25291630	0.14854787
7.50	3.51017750	0.30721581	1.35963430	1.36192470	0.22448348
10.00	3.52634790	0.39242782	1.48057100	1.48089520	0.31063722
12.50	3.52151720	0.47714443	1.61145620	1.60980930	0.40744561
15.00	3.48462340	0.56109051	1.75258270	1.74379090	0.51504350
17.50	3.40176500	0.64405242	1.90418180	1.89794050	0.63331415
20.00	3.25398240	0.72584350	2.06647340	2.05722110	0.75182250
22.50	3.01271470	0.80629127	2.23958650	2.22525410	0.89969857
25.00	2.62793090	0.88523215	2.42354690	2.40380010	1.04533260
27.50	1.98312900	0.96250900	2.61825740	2.58483750	1.19495570

AVERAGE DISTRIBUTION =1.99937400

0

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 10

TABLE:DISTRIBUTION-F(r)=ANNULAR FLOW

TH	DATA	r	F(r)	FC(r)	ERROR
0.00	1.40000000	0.00000000	1.00000000	1.00000010	-0.00000005
2.87	1.37244430	0.10000000	1.00000000	0.99999997	0.00000003
5.74	1.27543450	0.20000000	1.00000000	0.99999999	0.00000001
8.63	0.86864796	0.30000000	0.00000000	0.00000007	-0.00000007
11.54	0.93860307	0.40000000	0.00000000	0.00000002	0.00000002
14.48	1.06872580	0.50000000	0.00000000	0.00000015	0.00000015
17.46	1.60000000	0.60000000	1.00000000	0.99999999	0.00000001
20.49	1.42828570	0.70000000	1.00000000	1.00000010	-0.00000007
23.58	1.20000000	0.80000000	1.00000000	1.00000000	-0.00000003
26.74	0.87177989	0.90000001	1.00000000	1.00000020	-0.00000015

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 10

TABLE:DISTRIBUTION-F(r)=ANNULAR FLOW

TH	DATA	r	F(r)	FC(r)	ERROR
0.00	1.60000000	0.00000000	0.00000000	0.00000007	-0.00000007
2.87	1.61790220	0.10000000	0.00000000	0.00000022	0.00000022
5.74	1.67716990	0.20000000	0.00000000	0.00000010	0.00000010
8.63	1.80336690	0.30000000	0.00000000	0.00000000	0.00000000
11.54	2.28042000	0.40000000	1.00000000	1.00000020	-0.00000018
14.48	2.21510200	0.50000000	1.00000000	0.99999990	0.00000010
17.46	2.14169950	0.60000000	1.00000000	1.00000000	0.00000000
20.49	2.08197480	0.70000000	1.00000000	1.00000010	-0.00000013
23.58	2.40000000	0.80000000	2.00000000	2.00000010	-0.00000005
26.74	1.74355980	0.90000001	2.00000000	2.00000030	-0.00000033

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .000(PLEXI GLASS CYL)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.83478810	0.00000000	0.00000000	0.70291688	0.20587758
2.50	7.83280750	0.30533571	0.00000000	0.70404682	0.61705618
5.00	7.83439230	0.61009019	0.00000000	0.71773890	1.04310670
7.50	7.82923260	0.91368333	0.00000000	0.73290801	1.47084700
10.00	7.82084090	1.21553720	0.00000000	0.75374539	1.93632250
12.50	7.81237820	1.51507730	0.00000000	0.78445049	2.43221620
15.00	7.80343510	1.81173330	0.00000000	0.82922001	2.99155760
17.50	7.78239040	2.10494060	0.00000000	0.88705613	3.62595550
20.00	7.74413660	2.39414100	0.00000000	0.96427954	4.37133370
22.50	7.67275790	2.67878400	0.00000000	1.06743890	5.23142060
25.00	7.46336310	2.95832780	0.00000000	1.11605990	5.94543460

AVERAGE = 0.87259229

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .0000(AIR),[SCAN-1]

TH	DATA	r	F(r)	FI(r)	AREA PRODUCT
0.00	7.83478810	0.00000000	0.00000000	0.00000000	0-00000000
2.50	7.83280750	0.30533571	0.00000000	0.00000000	0-00000000
5.00	7.83439230	0.61009019	0.00000000	0.00000000	0-00000000
7.50	7.82923260	0.91368333	0.00000000	0.00000000	0-00000000
10.00	7.82084090	1.21553720	0.00000000	0.00000000	0-00000000
12.50	7.81237820	1.51507730	0.00000000	0.00000000	0-00000000
15.00	7.80343510	1.81173330	0.00000000	0.00000000	0-00000000
17.50	7.78239040	2.10494060	0.00000000	0.00000000	0-00000000
20.00	7.74413660	2.39414100	0.00000000	0.00000000	0-00000000
22.50	7.67275790	2.67878400	0.00000000	0.00000004	0-00000022
25.00	7.46336310	2.95832780	0.00000000	-0.00000004	-0-00000021

AVERAGE = 0.00000001

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .410(PINE),[SCAN-1]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.37462900	0.00000000	0.41000000	0.07793023	0.02283965
2.50	7.37400190	0.30533571	0.41000000	0.08012258	0.07022288
5.00	7.38398950	0.61009019	0.41000000	0.07943725	0.11544802
7.50	7.39079850	0.91368333	0.41000000	0.08112432	0.15380170
10.00	7.40549570	1.21553720	0.41000000	0.07813835	0.20078545
12.50	7.41997990	1.51507730	0.41000000	0.07995763	0.24793929
15.00	7.44716840	1.81173330	0.41000000	0.07809547	0.28175263
17.50	7.46737110	2.10494060	0.41000000	0.07883530	0.32225029
20.00	7.48549160	2.39414100	0.41000000	0.08025932	0.35408625
22.50	7.52617900	2.67878400	0.41000000	0.05493259	0.27719565
25.00	7.44424870	2.95832780	0.41000000	0.00703470	0.03747469

AVERAGE = 0.07497273

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .732(WALNUT),(SCAN-1)

TH	DATA	r	F(r)	FD(r)	AREA PRODUCT
0.00	7.02642690	0.00000000	0.73200000	0.10599582	0-03104515
2.50	7.01031190	0.30533571	0.73200000	0.13385921	0-41731983
5.00	7.01481440	0.61009019	0.73200000	0.14407254	0-09938415
7.50	7.03174130	0.91368333	0.73200000	0.14555721	0-09392119
10.00	7.05703700	1.21553720	0.73200000	0.14587318	0-37740502
12.50	7.09589330	1.51507730	0.73200000	0.14815833	0-45939523
15.00	7.15617660	1.81173330	0.73200000	0.14429205	0-62057541
17.50	7.21890970	2.10494060	0.73200000	0.14091448	0-57509757
20.00	7.28482090	2.39414100	0.73200000	0.13957193	0-53315056
22.50	7.38956400	2.67878400	0.73200000	0.10935403	0-51141579
25.00	7.44132040	2.95832780	0.73200000	0.00923139	0-04917659

AVERAGE = 0.13674189

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=1.000(WATER),(SCAN-1)

TH	DATA	r	F(r)	FO(r)	AREA PROJECT
0.00	5.60665020	0.00000000	1.00000000	0.21591659	0.05353278
2.50	5.61338420	0.30533571	1.00000000	0.21504170	0.13847157
5.00	5.63987580	0.61009019	1.00000000	0.21164357	0.30758555
7.50	5.67203300	0.91368333	1.00000000	0.21035612	0.42473933
10.00	5.71659480	1.21553720	1.00000000	0.20862529	0.53608314
12.50	5.77650700	1.51507730	1.00000000	0.20789197	0.61456812
15.00	5.85751410	1.81173330	1.00000000	0.20554002	0.74151671
17.50	5.95081480	2.10494060	1.00000000	0.20163338	0.82420660
20.00	7.05012260	2.39414100	1.00000000	0.20451380	0.92775124
22.50	7.21450440	2.67878400	1.00000000	0.17483279	0.85552428
25.00	7.41637850	2.95832780	1.00000000	0.01881350	0.10022222

AVERAGE = 0.20094483

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .900(10%-VOID),(SCAN-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.79570580	0.00000000	0.00000000	0.16225525	0.04752593
2.50	6.79570580	0.30533571	0.00000000	0.16555010	0.44510384
5.00	6.80461450	0.61009019	0.00000000	0.17405450	0.05297215
7.50	6.82762930	0.91368333	0.00000000	0.17327777	0.34987281
10.00	6.85856500	1.21553720	0.00000000	0.17247345	0.44318745
12.50	6.89871450	1.51507730	0.00000000	0.17403907	0.53958565
15.00	6.94793710	1.81173330	0.00000000	0.18221703	0.65740208
17.50	7.02731450	2.10494060	0.00000000	0.17781935	0.72685139
20.00	7.10496540	2.39414100	0.00000000	0.18121389	0.82205405
22.50	7.22256600	2.67878400	0.00000000	0.16715309	0.82750528
25.00	7.39756160	2.95832780	0.00000000	0.02275501	0.42121861

AVERAGE = 0.17502974

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .800(20%-VOID),[SCAN-1]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.90675480	0.00000000	0.80000000	0.15031400	0.04402552
2.50	6.91274290	0.30533571	0.80000000	0.14327039	0.12555862
5.00	6.92559520	0.61009019	0.80000000	0.14215797	0.20660151
7.50	6.93049480	0.91368333	0.80000000	0.15133259	0.30556328
10.00	6.95463890	1.21553720	0.80000000	0.15053618	0.33681747
12.50	6.98193470	1.51507730	0.80000000	0.15585451	0.43322520
15.00	7.02553830	1.81173330	0.80000000	0.16149930	0.53255571
17.50	7.08840880	2.10494060	0.80000000	0.15870599	0.64873289
20.00	7.13966040	2.39414100	0.80000000	0.16953455	0.75929967
22.50	7.23705910	2.67878400	0.80000000	0.16172950	0.80055560
25.00	7.39633530	2.95832780	0.80000000	0.02469450	0.43155050

AVERAGE = 0.15838280

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .700(30%-VOID),(SCAN-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.03526860	0.00000000	0.70000000	0.11943416	0.03499575
2.50	7.03790600	0.30533571	0.70000000	0.11495922	0.10975508
5.00	7.03966040	0.61009019	0.70000000	0.12471928	0.18125744
7.50	7.05444970	0.91368333	0.70000000	0.12108043	0.21447385
10.00	7.06561340	1.21553720	0.70000000	0.12147210	0.31213445
12.50	7.06987420	1.51507730	0.70000000	0.13459726	0.41731621
15.00	7.09754880	1.81173330	0.70000000	0.14390337	0.51917595
17.50	7.14598450	2.10494060	0.70000000	0.14363882	0.58714370
20.00	7.18083120	2.39414100	0.70000000	0.15835938	0.71842287
22.50	7.26542970	2.67878400	0.70000000	0.15251479	0.75503717
25.00	7.40610340	2.95832780	0.70000000	0.02191206	0.41688794

AVERAGE = 0.14078268

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .600(40%-VOID),(SCAN-1)

TH	DATA	r	F(r)	FI(r)	AREA PRODUCT
0.00	7.20191630	0.00000000	0.60000000	0.09414783	0.02757499
2.50	7.20637730	0.30533571	0.60000000	0.08655477	0.07586900
5.00	7.21597500	0.61009019	0.60000000	0.08085652	0.11751082
7.50	7.20414930	0.91368333	0.60000000	0.09385534	0.13950783
10.00	7.21303160	1.21553720	0.60000000	0.08994829	0.23113091
12.50	7.20191630	1.51507730	0.60000000	0.10505403	0.32882005
15.00	7.21817680	1.81173330	0.60000000	0.11281404	0.40711839
17.50	7.23273310	2.10494060	0.60000000	0.12403809	0.50702300
20.00	7.25981960	2.39414100	0.60000000	0.13450405	0.61016077
22.50	7.31121840	2.67878400	0.60000000	0.13652157	0.67585191
25.00	7.41878090	2.95832780	0.60000000	0.01725653	0.09192810

AVERAGE = 0.11534759

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .900(10%-VOID),ISCAN=2

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.81344460	0.00000000	0.00000000	0.17971734	0.05263748
2.50	6.82328610	0.30533571	0.00000000	0.16831558	0.14751893
5.00	6.83733280	0.61009019	0.00000000	0.17263438	0.25089432
7.50	6.86589110	0.91368333	0.00000000	0.16513034	0.33342197
10.00	6.88243750	1.21553720	0.00000000	0.17513931	0.45003762
12.50	6.92755790	1.51507730	0.00000000	0.17583924	0.54828927
15.00	6.99668150	1.81173330	0.00000000	0.17221157	0.62130439
17.50	7.06133440	2.10494060	0.00000000	0.17440815	0.71291763
20.00	7.13409380	2.39414100	0.00000000	0.18225200	0.82575336
22.50	7.26892010	2.67878400	0.00000000	0.16049315	0.79453470
25.00	7.45240250	2.95832780	0.00000000	0.00785883	0.04135489

AVERAGE = 0.17238833

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .800(20%-VOID),(SCAN-2)

TH	DATA	r	F(r)	FI(r)	AREA PRODUCT
0.00	6.94697600	0.00000000	0.80000000	0.14631443	0.04285409
2.50	6.95463890	0.30533571	0.80000000	0.13413016	0.11755730
5.00	6.95368420	0.61009019	0.80000000	0.15112427	0.21953244
7.50	6.98193470	0.91368333	0.80000000	0.14044777	0.23358479
10.00	6.99393300	1.21553720	0.80000000	0.14751720	0.37005991
12.50	7.02375890	1.51507730	0.80000000	0.15122101	0.45386008
15.00	7.06987420	1.81173330	0.80000000	0.15352707	0.55369151
17.50	7.11963560	2.10494060	0.80000000	0.15805852	0.64517713
20.00	7.17778240	2.39414100	0.80000000	0.16650007	0.75575034
22.50	7.28138570	2.67878400	0.80000000	0.15545470	0.77159154
25.00	7.45876270	2.95832780	0.80000000	0.00514035	0.02738324

AVERAGE = 0.15428374

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .700(30%-VOID),(SCAN-2)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.08002650	0.00000000	0.70000000	0.11675327	0.03419591
2.50	7.08422640	0.30533571	0.70000000	0.10950629	0.09597591
5.00	7.08673790	0.61009019	0.70000000	0.11686852	0.15984775
7.50	7.09257370	0.91368333	0.70000000	0.12175549	0.21584178
10.00	7.11069610	1.21553720	0.70000000	0.11950336	0.30707561
12.50	7.12769370	1.51507730	0.70000000	0.12324765	0.33212873
15.00	7.14834580	1.81173330	0.70000000	0.13364792	0.43217458
17.50	7.18690100	2.10494060	0.70000000	0.13645315	0.55781215
20.00	7.21450440	2.39414100	0.70000000	0.15399101	0.69856090
22.50	7.29709100	2.67878400	0.70000000	0.15004031	0.71278702
25.00	7.45991480	2.95832780	0.70000000	0.00365615	0.01953165

5.00	7.08673790	0.61009019	0.70000000	0.11686852	0.15984775
7.50	7.09257370	0.91368333	0.70000000	0.12175549	0.21584178
10.00	7.11069610	1.21553720	0.70000000	0.11950336	0.30707561
12.50	7.12769370	1.51507730	0.70000000	0.12324765	0.33212876
15.00	7.14834580	1.81173330	0.70000000	0.13364792	0.43217458
17.50	7.18690100	2.10494060	0.70000000	0.13645315	0.55781215
20.00	7.21450440	2.39414100	0.70000000	0.15399101	0.69856090
22.50	7.29709100	2.67878400	0.70000000	0.15004031	0.71278702

0

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .600(40%-VOID),[SCAN-2]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.23489840	0.00000000	0.60000000	0.08345588	0.02444342
2.50	7.23849680	0.30533571	0.60000000	0.07443661	0.05528319
5.00	7.23633930	0.61009019	0.60000000	0.08080825	0.41744052
7.50	7.22911390	0.91368333	0.60000000	0.08963717	0.48099037
10.00	7.23417720	1.21553720	0.60000000	0.08873375	0.22801006
12.50	7.22983880	1.51507730	0.60000000	0.09875256	0.30618185
15.00	7.23489840	1.81173330	0.60000000	0.11065010	0.39923919
17.50	7.24636810	2.10494050	0.60000000	0.12320879	0.60363311
20.00	7.26892010	2.39414100	0.60000000	0.13779803	0.62510348
22.50	7.32974970	2.67878400	0.60000000	0.13857557	0.63502989
25.00	7.46679950	2.95832780	0.60000000	0.00200880	0.01070109

AVERAGE = 0.11407297

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .900(10%-VOID),ISCAN=3)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.82654520	0.00000000	0.00000000	0.17461503	0.05114305
2.50	6.82871210	0.30533571	0.00000000	0.18001474	0.15776985
5.00	6.86171130	0.61009019	0.00000000	0.16392059	0.23822955
7.50	6.88038410	0.91363333	0.00000000	0.16385256	0.33084215
10.00	6.89770500	1.21553720	0.00000000	0.17375234	0.44649937
12.50	6.94985650	1.51507730	0.00000000	0.16811101	0.52122743
15.00	6.99942250	1.81173330	0.00000000	0.17414700	0.62828722
17.50	7.06475900	2.10494060	0.00000000	0.17835345	0.72904463
20.00	7.15383380	2.39414100	0.00000000	0.17774092	0.80629940
22.50	7.28207370	2.67878400	0.00000000	0.15825342	0.73344671
25.00	7.46336310	2.95832780	0.00000000	0.00635798	0.03392293

AVERAGE = 0.17058234

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .800(20%-VOID),[SCAN-3]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.98193470	0.00000000	0.80000000	0.12525707	0.03668952
2.50	6.97821380	0.30533571	0.80000000	0.13255131	0.11617397
5.00	6.98379000	0.61009019	0.80000000	0.13952649	0.20277711
7.50	6.99209640	0.91368333	0.80000000	0.14795989	0.29875235
10.00	7.02820150	1.21553720	0.80000000	0.13712249	0.35234956
12.50	7.04925480	1.51507730	0.80000000	0.14253107	0.44207196
15.00	7.08002650	1.81173330	0.80000000	0.15209144	0.51871508
17.50	7.12367280	2.10494060	0.80000000	0.16137094	0.65962622
20.00	7.19293420	2.39414100	0.80000000	0.16515795	0.71926374
22.50	7.29979740	2.67878400	0.80000000	0.15270638	0.75598565
25.00	7.47250070	2.95832780	0.80000000	0.00284900	0.01517594

AVERAGE = 0.15139163

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .700(30%-VOID),(SCAN-3)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.13169850	0.00000000	0.70000000	0.10294429	0.03015139
2.50	7.13009650	0.30533571	0.70000000	0.10664910	0.09347175
5.00	7.13886700	0.61009019	0.70000000	0.10815592	0.15718553
7.50	7.15148550	0.91368333	0.70000000	0.10312184	0.20821785
10.00	7.15226890	1.21553720	0.70000000	0.10899712	0.28007879
12.50	7.15851400	1.51507730	0.70000000	0.11770725	0.35495079
15.00	7.17625460	1.81473330	0.70000000	0.12874340	0.45448008
17.50	7.21376830	2.10494060	0.70000000	0.13044102	0.53319588
20.00	7.23417720	2.39414100	0.70000000	0.15039425	0.63224468
22.50	7.31588350	2.67878400	0.70000000	0.14517138	0.71858300
25.00	7.46737110	2.95832780	0.70000000	0.00415240	0.02217360

AVERAGE = 0.12848704

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .600(40%-VOID),(SCAN-3)

TH	DATA	r	F(r)	FC(r)	AREA PROJC
0.00	7.26262860	0.00000000	0.60000000	0.06533129	0.01913490
2.50	7.25205400	0.30533571	0.60000000	0.08245250	0.07225492
5.00	7.26122510	0.61009019	0.60000000	0.07834109	0.11385486
7.50	7.25911620	0.91368333	0.60000000	0.07977132	0.15105981
10.00	7.24422750	1.21553720	0.60000000	0.09413242	0.24201097
12.50	7.25205400	1.51507730	0.60000000	0.09880815	0.30635431
15.00	7.25682730	1.81173330	0.60000000	0.10452031	0.37708805
17.50	7.27378630	2.10494060	0.60000000	0.11814730	0.48294355
20.00	7.29437730	2.39414100	0.60000000	0.13178257	0.59781558
22.50	7.34601020	2.67878400	0.60000000	0.13561950	0.57139561
25.00	7.47703850	2.95832780	0.60000000	0.00240755	0.01282570

AVERAGE = 0.11071145

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .900(10%-VOID),(SCAN-4)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.78219210	0.00000000	0.90000000	0.19180632	0.05617822
2.50	6.79682370	0.30533571	0.90000000	0.17224323	0.15096120
5.00	6.80903930	0.61009019	0.90000000	0.17840372	0.25927630
7.50	6.83195360	0.91368333	0.90000000	0.18000635	0.35345980
10.00	6.86901440	1.21553720	0.90000000	0.17817436	0.45763549
12.50	6.92165820	1.51507730	0.90000000	0.17345011	0.53778130
15.00	6.97541390	1.81173330	0.90000000	0.17849634	0.64398038
17.50	7.04838650	2.10494060	0.90000000	0.17795808	0.72746935
20.00	7.12769370	2.39414100	0.90000000	0.18281915	0.82933630
22.50	7.25911620	2.67878400	0.90000000	0.16385384	0.81122155
25.00	7.45356190	2.95832780	0.90000000	0.00565119	0.03010155

AVERAGE = 0.17594574

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .800(20%-VOID),(SCAN-4)

TH	DATA	r	F(r)	FD(r)	AREA PRODUCT
0.00	6.94119010	0.00000000	0.80000000	0.14371951	0.04209109
2.50	6.94505110	0.30533571	0.80000000	0.13965465	0.12240797
5.00	6.95559260	0.61009019	0.80000000	0.14093498	0.20489677
7.50	6.95939850	0.91368333	0.80000000	0.15281086	0.30854716
10.00	6.99117690	1.21553720	0.80000000	0.14755632	0.37914308
12.50	7.02286810	1.51507730	0.80000000	0.14954505	0.45356375
15.00	7.06561340	1.81173330	0.80000000	0.15299280	0.53196599
17.50	7.11069610	2.10494060	0.80000000	0.16054643	0.63625592
20.00	7.17242460	2.39414100	0.80000000	0.16827392	0.75335354
22.50	7.28344820	2.67878400	0.80000000	0.15359721	0.75039579
25.00	7.45124170	2.95832780	0.80000000	0.00613330	0.03257282

AVERAGE = 0.15458759

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .700(30%-VOID),[SCAN-4]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.09837570	0.00000000	0.70000000	0.10523605	0.03082263
2.50	7.09672140	0.30533571	0.70000000	0.10874150	0.09530585
5.00	7.10742550	0.61009019	0.70000000	0.10447589	0.15183725
7.50	7.09754880	0.91368333	0.70000000	0.12154485	0.21541657
10.00	7.12205990	1.21553720	0.70000000	0.11285507	0.23999215
12.50	7.12929760	1.51507730	0.70000000	0.12061132	0.37395542
15.00	7.14203660	1.81173330	0.70000000	0.13503900	0.43737383
17.50	7.17548970	2.10494060	0.70000000	0.14353125	0.53670399
20.00	7.22402480	2.39414100	0.70000000	0.15095373	0.63482505
22.50	7.30384330	2.67878400	0.70000000	0.14584433	0.72596505
25.00	7.45876270	2.95832780	0.70000000	0.00341345	0.01834358

AVERAGE = 0.13369373

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .600(40%-VOID),(SCAN-4)

TH	DATA	r	F(r)	FD(r)	AREA PRODUCT
0.00	7.25981960	0.00000000	0.60000000	0.08490492	0.02486783
2.50	7.27100860	0.30533571	0.60000000	0.06025631	0.05282036
5.00	7.25417790	0.61009019	0.60000000	0.08288409	0.12045733
7.50	7.25981960	0.91368333	0.60000000	0.07965293	0.15063075
10.00	7.25347040	1.21553720	0.60000000	0.08645875	0.22218995
12.50	7.25700270	1.51507730	0.60000000	0.08937055	0.27709299
15.00	7.25134500	1.81173330	0.60000000	0.10493695	0.37859125
17.50	7.25559130	2.10494060	0.60000000	0.11991254	0.49028225
20.00	7.27655640	2.39414100	0.60000000	0.13345372	0.60544143
22.50	7.32909380	2.67878400	0.60000000	0.13633327	0.67492921
25.00	7.45472000	2.95832780	0.60000000	0.00562018	0.02993935

AVERAGE = 0.10938548

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .900(10%-VOID),(SCAN=5)

TH	DATA	r	F(r)	FO(r)	AREA PRODUCT
0.00	5.78671690	0.00000000	0.00000000	0.18537534	0.05129464
2.50	5.79570580	0.30533571	0.00000000	0.17601710	0.15426382
5.00	5.81124440	0.61009019	0.00000000	0.17951347	0.25089171
7.50	5.83518460	0.91368338	0.00000000	0.18132293	0.35611715
10.00	5.87935580	1.21553720	0.00000000	0.17281035	0.44405444
12.50	5.91869520	1.51507730	0.00000000	0.17725934	0.54952129
15.00	5.98471640	1.81173330	0.00000000	0.17425510	0.62871311
17.50	7.04403290	2.10494060	0.00000000	0.18210635	0.71433723
20.00	7.13727840	2.39414100	0.00000000	0.17953604	0.81465955
22.50	7.26052260	2.67878400	0.00000000	0.16123403	0.81330175
25.00	7.45529850	2.95832780	0.00000000	0.00564330	0.03550275

AVERAGE = 0.17558459

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .800(20%-VOID),(SCAN-5)

TH	DATA	r	F(r)	FZ(r)	AREA PRODUCT
0.00	5.91373740	0.00000000	0.80000000	0.14220525	0.04165055
2.50	5.90675480	0.30533571	0.80000000	0.16052200	0.11058827
5.00	5.93439720	0.61009019	0.80000000	0.14572725	0.21324217
7.50	5.94215670	0.91368333	0.80000000	0.15553301	0.31414453
10.00	5.97073010	1.21553720	0.80000000	0.15434175	0.39659529
12.50	7.01031190	1.51507730	0.80000000	0.15245032	0.47257154
15.00	7.05531290	1.81173330	0.80000000	0.15497335	0.55911415
17.50	7.10002720	2.10494060	0.80000000	0.16314334	0.65687105
20.00	7.16006920	2.39414100	0.80000000	0.17359530	0.73749360
22.50	7.28069720	2.67878400	0.80000000	0.15507232	0.77254900
25.00	7.45529850	2.95832780	0.80000000	0.00617887	0.93291557

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .700(30%-VOID),(SCAN=5)

TH	DATA	r	F(r)	FC(r)	AREA PROJECT
0.00	7.04228620	0.00000000	0.70000000	0.12787937	0.03745453
2.50	7.05012260	0.30533571	0.70000000	0.11460425	0.10044393
5.00	7.05098940	0.61009019	0.70000000	0.12497253	0.13162561
7.50	7.06390400	0.91368333	0.70000000	0.12290854	0.21817007
10.00	7.06817200	1.21553720	0.70000000	0.13343534	0.31300365
12.50	7.10496540	1.51607730	0.70000000	0.12573674	0.39000121
15.00	7.11801620	1.81173330	0.70000000	0.14232995	0.51319763
17.50	7.16239750	2.10494050	0.70000000	0.14593954	0.50675302
20.00	7.20934030	2.39414100	0.70000000	0.15562495	0.70597298
22.50	7.29573510	2.67878400	0.70000000	0.14983935	0.74179257
25.00	7.45529850	2.95832780	0.70000000	0.00553539	0.02975571

AVERAGE = 0.14034507

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .600(40%-VOID),(SCAN-3)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.22037390	0.00000000	0.60000000	0.08853619	0.02594601
2.50	7.22183590	0.30533571	0.60000000	0.08562001	0.07504097
5.00	7.22911390	0.61009019	0.60000000	0.08343307	0.12125523
7.50	7.22402480	0.91368333	0.60000000	0.08992902	0.43157965
10.00	7.22256600	1.21553720	0.60000000	0.09493899	0.24395750
12.50	7.22183590	1.51507730	0.60000000	0.10637319	0.32980950
15.00	7.24565510	1.81173330	0.60000000	0.10814475	0.39016441
17.50	7.25205400	2.10494050	0.60000000	0.12417015	0.50756285
20.00	7.28619170	2.39414100	0.60000000	0.13115491	0.59495781
22.50	7.33367640	2.67878400	0.60000000	0.13641517	0.67533962
25.00	7.46106550	2.95832780	0.60000000	0.00520712	0.92771053

AVERAGE = 0.11441008

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .000(N=N+SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.85448570	0.00000000	0.00000000	0.70452922	0.20534982
2.50	7.85252450	0.30533571	0.00000000	0.70569740	0.61850287
5.00	7.85409380	0.61009019	0.00000000	0.71937252	1.04548110
7.50	7.84898440	0.91368330	0.00000000	0.73453278	1.43322860
10.00	7.84067500	1.21553720	0.00000000	0.75549025	1.94130530
12.50	7.83229560	1.51507730	0.00000000	0.78523343	2.43735320
15.00	7.82344080	1.81173330	0.00000000	0.83112092	2.99851570
17.50	7.80260560	2.10494050	0.00000000	0.88910095	3.63432410
20.00	7.76473830	2.39414100	0.00000000	0.96557478	4.33474540
22.50	7.69410010	2.67873400	0.00000000	1.07023090	5.29825710
25.00	7.48703320	2.95832780	0.00000000	1.12105440	5.97204080

AVERAGE = 0.87457583

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=1.000(N=N+SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.64275140	0.00000000	1.00000000	0.21381929	0.05262551
2.50	5.64936620	0.30533571	1.00000000	0.21195507	0.13577508
5.00	5.67539270	0.61009019	1.00000000	0.20863029	0.30327994
7.50	5.70699310	0.91368333	1.00000000	0.20744534	0.41885205
10.00	5.75079760	1.21553720	1.00000000	0.20577591	0.52875395
12.50	5.80971700	1.51507730	1.00000000	0.20405973	0.63268529
15.00	5.88942660	1.81173330	1.00000000	0.20287584	0.73193487
17.50	5.98129480	2.10494060	1.00000000	0.19910170	0.81385597
20.00	7.07914720	2.39414100	1.00000000	0.20200541	0.91637573
22.50	7.24126940	2.67878400	1.00000000	0.17275452	0.85528564
25.00	7.44060460	2.95832780	1.00000000	0.01853307	0.09899421

AVERAGE = 0.49820285

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .732(N=N+SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.05579240	0.00000000	0.73200000	0.10475408	0.03068145
2.50	7.03991160	0.30533571	0.73200000	0.13220204	0.11586741
5.00	7.04434840	0.61009019	0.73200000	0.14229338	0.20679902
7.50	7.05103000	0.91368333	0.73200000	0.14377745	0.29030742
10.00	7.08596300	1.21553720	0.73200000	0.14507490	0.37278419
12.50	7.12427040	1.51507730	0.73200000	0.14532345	0.45057472
15.00	7.18372280	1.81173330	0.73200000	0.14253595	0.51442120
17.50	7.24561650	2.10494060	0.73200000	0.13923007	0.55932572
20.00	7.31067300	2.39414100	0.73200000	0.13797133	0.62589193
22.50	7.41411300	2.67878400	0.73200000	0.10812791	0.53529524
25.00	7.46524980	2.95832780	0.73200000	0.00911480	0.04855562

AVERAGE = 0.13600305

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .410(N=N+SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.39935980	0.00000000	0.41000000	0.07712071	0.02258791
2.50	7.39874040	0.30533571	0.41000000	0.07923079	0.05944119
5.00	7.40860620	0.61009019	0.41000000	0.07856502	0.11418038
7.50	7.41533260	0.91368333	0.41000000	0.08023825	0.15200251
10.00	7.42985230	1.21553720	0.41000000	0.07723340	0.19858725
12.50	7.44416290	1.51507730	0.41000000	0.07804353	0.24197361
15.00	7.47102870	1.81173330	0.41000000	0.07725238	0.27871092
17.50	7.49099440	2.10494050	0.41000000	0.07793655	0.31878094
20.00	7.50890430	2.39414100	0.41000000	0.07933985	0.35014210
22.50	7.54912560	2.67878400	0.41000000	0.05433311	0.25922813
25.00	7.46814350	2.95832780	0.41000000	0.00694129	0.03697705

AVERAGE = 0.07403847

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .000(N=N-SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.81469470	0.00000000	0.00000000	0.70127448	0.20539554
2.50	7.81269400	0.30533671	0.00000000	0.70235528	0.61558241
5.00	7.81429490	0.61009018	0.00000000	0.71507551	1.04058930
7.50	7.80908270	0.91363338	0.00000000	0.73120251	1.47540350
10.00	7.80060550	1.21553720	0.00000000	0.75195829	1.93225520
12.50	7.79205610	1.51507730	0.00000000	0.78260335	2.42545970
15.00	7.78302100	1.81173330	0.00000000	0.82723455	2.93457490
17.50	7.76175800	2.10494050	0.00000000	0.88197417	3.51745530
20.00	7.72310170	2.39414100	0.00000000	0.95194135	4.85372370
22.50	7.65095030	2.67873400	0.00000000	1.05453090	5.27029530
25.00	7.43911900	2.95832730	0.00000000	1.11093250	5.91805790

AVERAGE = 0.87035954

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=1.000(N=N-SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.56919680	0.00000000	1.00000000	0.22020832	0.05449590
2.50	6.57605910	0.30533671	1.00000000	0.21797042	0.19103842
5.00	6.60305100	0.61009019	1.00000000	0.21373905	0.31070464
7.50	6.63580620	0.91368333	1.00000000	0.21344325	0.43097271
10.00	6.68118060	1.21553720	1.00000000	0.21163853	0.54381311
12.50	6.74215610	1.51507730	1.00000000	0.21083654	0.65369464
15.00	6.82454940	1.81173330	1.00000000	0.20830335	0.75151525
17.50	6.91937650	2.10494060	1.00000000	0.20430245	0.83611475
20.00	7.02023020	2.39414100	1.00000000	0.20715158	0.93871717
22.50	7.18700340	2.67878400	1.00000000	0.17595425	0.87602680
25.00	7.39155100	2.95832780	1.00000000	0.01904435	0.10145420

AVERAGE = 0.20367303

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .732(N=N-SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.99617270	0.00000000	0.73200000	0.10729834	0.03142580
2.50	5.97980930	0.30533571	0.73200000	0.13525252	0.11854983
5.00	5.98438140	0.61009019	0.73200000	0.14494556	0.21065278
7.50	7.00156870	0.91368338	0.73200000	0.14745103	0.29772489
10.00	7.02724930	1.21553720	0.73200000	0.14875555	0.38224197
12.50	7.06668720	1.51507730	0.73200000	0.15005542	0.45524514
15.00	7.12785020	1.81173330	0.73200000	0.14503833	0.52685044
17.50	7.19147010	2.10494050	0.73200000	0.14262908	0.53301625
20.00	7.25828270	2.39414100	0.73200000	0.14124878	0.64075741
22.50	7.36439700	2.67878400	0.73200000	0.11060892	0.64757855
25.00	7.41680440	2.95832780	0.73200000	0.00934298	0.04977115

AVERAGE = 0.13836802

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .410(N=N-SQRT(N))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.34927110	0.00000000	0.41000000	0.07887925	0.02310297
2.50	7.34863590	0.30533671	0.41000000	0.08071450	0.07074165
5.00	7.35875140	0.61009019	0.41000000	0.07934951	0.11532065
7.50	7.36564740	0.91368338	0.41000000	0.08205640	0.15568371
10.00	7.38053100	1.21553720	0.41000000	0.07902882	0.20305944
12.50	7.39519760	1.51507730	0.41000000	0.08083137	0.25077233
15.00	7.42272480	1.81173330	0.41000000	0.07892672	0.23475159
17.50	7.44317620	2.10494060	0.41000000	0.07972231	0.32587605
20.00	7.46151760	2.39414100	0.41000000	0.08115757	0.36820695
22.50	7.50269340	2.67878400	0.41000000	0.05555947	0.27505179
25.00	7.41976890	2.95832780	0.41000000	0.00712210	0.03794026

AVERAGE = 0.07574545

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .000(N=N+3.0*SQRT(N))

TH	DATA	r	F(r)	FI(r)	AREA PRODUCT
0.00	7.89275380	0.00000000	0.00000000	0.70765935	0.20726963
2.50	7.89082900	0.30533671	0.00000000	0.70391005	0.62131853
5.00	7.89236920	0.61009019	0.00000000	0.72255492	1.05010500
7.50	7.88735490	0.91368333	0.00000000	0.73784432	1.43981510
10.00	7.87920060	1.21553720	0.00000000	0.75883779	1.95003660
12.50	7.87097830	1.51507730	0.00000000	0.78983234	2.44887320
15.00	7.86229010	1.81473830	0.00000000	0.83482875	3.01187480
17.50	7.84184990	2.10494050	0.00000000	0.89303414	3.65060590
20.00	7.80471030	2.39414100	0.00000000	0.97104301	4.40501490
22.50	7.73546480	2.67873400	0.00000000	1.07565430	5.82515830
25.00	7.53275630	2.95832780	0.00000000	1.13065390	5.02312580

AVERAGE = 0.87373099

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=1.000(N=N+(3.0*SQRT(N)))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.71126440	0.00000000	1.00000000	0.20814235	0.05096304
2.50	5.71766450	0.30533571	1.00000000	0.20532518	0.13083292
5.00	5.74285220	0.61009019	1.00000000	0.20323034	0.29537321
7.50	5.77344740	0.91368338	1.00000000	0.20209440	0.40805773
10.00	5.81588190	1.21553720	1.00000000	0.20043380	0.51516277
12.50	5.87300090	1.51507730	1.00000000	0.19997841	0.62003217
15.00	5.95035000	1.81173830	1.00000000	0.19794915	0.71415043
17.50	7.03960160	2.10494050	1.00000000	0.19435623	0.79445817
20.00	7.13478540	2.39414100	1.00000000	0.19734699	0.89523987
22.50	7.29274140	2.67878400	1.00000000	0.16889919	0.83614948
25.00	7.48736400	2.95832780	1.00000000	0.01813153	0.09685515

AVERAGE = 0.49361037

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .732(N=N+(3.0*SQRT(N)))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.11205670	0.00000000	0.73200000	0.10243928	0.03000348
2.50	7.09660540	0.30533571	0.73200000	0.12912559	0.11317118
5.00	7.10092200	0.61009019	0.73200000	0.13899155	0.20199969
7.50	7.11715330	0.91368333	0.73200000	0.14045235	0.23359455
10.00	7.14141960	1.21553720	0.73200000	0.14163359	0.35406988
12.50	7.17871770	1.51507730	0.73200000	0.14303438	0.44347399
15.00	7.23663810	1.81173330	0.73200000	0.13940717	0.50295280
17.50	7.29698110	2.10494050	0.73200000	0.13613025	0.55665580
20.00	7.36045460	2.39414100	0.73200000	0.13493328	0.61233473
22.50	7.46147390	2.67878400	0.73200000	0.10580572	0.52380005
25.00	7.51145630	2.95832780	0.73200000	0.00892555	0.04754804

AVERAGE = 0.13210230

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .410(N=N+(3.0*SQRT(N)))

TH	DATA	r	F(r)	FD(r)	AREA PRODUCT
0.00	7.44705900	0.00000000	0.41000000	0.07551100	0.02211644
2.50	7.44645370	0.30533671	0.41000000	0.07755154	0.05797819
5.00	7.45609310	0.61009019	0.41000000	0.07693479	0.11180577
7.50	7.46266570	0.91368333	0.41000000	0.07855378	0.15853160
10.00	7.47685500	1.21553720	0.41000000	0.07563051	0.19434000
12.50	7.49084220	1.51507730	0.41000000	0.07745604	0.24014899
15.00	7.51710650	1.81173830	0.41000000	0.07567122	0.27300641
17.50	7.53663000	2.10494050	0.41000000	0.07634447	0.31205865
20.00	7.55414690	2.39414100	0.41000000	0.07775950	0.35274521
22.50	7.59349710	2.67878400	0.41000000	0.05324873	0.25361229
25.00	7.61428540	2.95832780	0.41000000	0.00673554	0.03620119

AVERAGE = 0.07251347

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .000(N=N-3.0*SQRT(N))

TH	DATA	r	F(r)	FCK(r)	AREA PRODUCT
0.00	7.77325460	0.00000000	0.00000000	0.69789696	0.20440729
2.50	7.77121110	0.30533671	0.00000000	0.69890459	0.61254940
5.00	7.77284620	0.61009019	0.00000000	0.71265493	1.03671810
7.50	7.76752250	0.91368338	0.00000000	0.72769571	1.45932260
10.00	7.75886310	1.21553720	0.00000000	0.74831270	1.92235280
12.50	7.75012930	1.51507730	0.00000000	0.77373447	2.41451770
15.00	7.74089830	1.81173330	0.00000000	0.82330500	2.97031750
17.50	7.71917070	2.10494050	0.00000000	0.88069361	3.69995800
20.00	7.67965590	2.39414100	0.00000000	0.95713019	4.84191020
22.50	7.60585420	2.67878400	0.00000000	1.05869770	5.24117100
25.00	7.38879300	2.95832780	0.00000000	1.40019750	5.85038070

AVERAGE = 0.85510145

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)=1.000(N=N-(3.0*SQRT(N)))

TH	DATA	r	F(r)	FCI(r)	AREA PROD
0.00	5.48979930	0.00000000	1.00000000	0.22745853	0.05662041
2.50	5.49694990	0.30533671	1.00000000	0.22551530	0.19765107
5.00	5.52506500	0.61009019	1.00000000	0.22170158	0.32220409
7.50	5.55916070	0.91368338	1.00000000	0.22021828	0.44465245
10.00	5.60635200	1.21553720	1.00000000	0.21825694	0.55083003
12.50	5.66969790	1.51507730	1.00000000	0.21722104	0.67349285
15.00	5.75516940	1.81173830	1.00000000	0.21443192	0.77380725
17.50	5.85336990	2.10494060	1.00000000	0.21009645	0.85879853
20.00	5.95762340	2.39414100	1.00000000	0.21285523	0.95563542
22.50	7.12962290	2.67878400	1.00000000	0.18163951	0.89945903
25.00	7.38996640	2.95832780	1.00000000	0.01954387	0.10411249

AVERAGE = 0.20961307

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .732(N=N-(3.0*SQRT(N)))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	6.93277200	0.00000000	0.73200000	0.11011056	0.03225035
2.50	6.91586240	0.30533671	0.73200000	0.13933815	0.12216563
5.00	6.92058770	0.61009019	0.73200000	0.15000580	0.21800694
7.50	6.93834690	0.91368333	0.73200000	0.15153878	0.30596855
10.00	6.95487220	1.21553720	0.73200000	0.15285426	0.39279969
12.50	7.00558440	1.51507730	0.73200000	0.15403938	0.47775497
15.00	7.06867010	1.81173330	0.73200000	0.14995195	0.64099522
17.50	7.13422320	2.10494050	0.73200000	0.14532414	0.59812031
20.00	7.20299520	2.39414100	0.73200000	0.14485959	0.65713785
22.50	7.34207950	2.67878400	0.73200000	0.11344207	0.65160441
25.00	7.36589190	2.95832780	0.73200000	0.00958333	0.05105419

AVERAGE = 0.14209514

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE:DISTRIBUTION-F(r)= .410(N=N-(3.0*SQRT(N)))

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.29654050	0.00000000	0.41000000	0.08080912	0.02365821
2.50	7.29588790	0.30533671	0.41000000	0.08305154	0.07279862
5.00	7.30628010	0.61009019	0.41000000	0.08230689	0.11961853
7.50	7.31336380	0.91368338	0.41000000	0.08405655	0.15972229
10.00	7.32865020	1.21553720	0.41000000	0.08095318	0.20801713
12.50	7.34371070	1.51507730	0.41000000	0.08279255	0.25659793
15.00	7.37196860	1.81173330	0.41000000	0.08085470	0.29174342
17.50	7.39295600	2.10494060	0.41000000	0.08162218	0.33364205
20.00	7.41177310	2.39414100	0.41000000	0.08311650	0.37704303
22.50	7.45399980	2.67878400	0.41000000	0.05695304	0.28195078
25.00	7.36893480	2.95832780	0.41000000	0.00731417	0.03894743

AVERAGE = 0.07754893

```
C**       THIS PROGRAM CALCULATES THE AVERAGE DISTRIBUTION
C**       FOR SIMULATED DATA WITH RADIUS=1 AND THE DISTANCE
C**       OF SOURCE FROM THE CENTRE=2.0.THIS PROGRAM FIRST
C**       CALCULATES THE NUMERICAL INTEGRATION BY SIMPSON'S
C**       1/3 RULE ALONG VARIOUS CHORDS SO WE HAVE THE DATA
C**       VECTOR(I) ,THEN IT CALCULATES THE VARIOUS S(K,J)
C**       FOR VARIOUS CHORDS AND THEN RECONSTRUCTS THE DISTRIBUTION
C**       BY BACK SUBSTITUTION
C**
          INTEGER N
          REAL ID,I1,DAT(310),THH(310),S(310,310),F(310),FX(310)
          REAL T(310),AREA(310),ERR(310)
          COMMON TH,D,E
          READ(20,*)R,D,TH,DTH,E,N
          THH(1)=0.0
          THH(N+1)=(ASIN(R/D))*(180./3.141593)
          TYPE *,THH(N+1)
          DO 10 I=2,N
          THH(I)=THH(I-1)+DTH
          TYPE *,THH(I)
10        CONTINUE
          T(1)=0.0
          DO 20 I=1,N+1
          T(I)=(D*SIND(THH(I)))
20        CONTINUE
          DO 25 I=1,N
          AREA(I)=3.141593*((T(I+1)**2)-(T(I)**2))
25        CONTINUE
          SUMM1=0.0
          DO 27 I=1,N
          SUMM1=SUMM1+AREA(I)
27        CONTINUE
          DO 28 I=1,N
          R1=T(I)
          R1=R1*R1
          R2=T(I+1)
          R2=R2*R2
          RR=(R1+R2)/2.0
          RR=SQRT(RR)
          T(I)=RR
28        CONTINUE
          DO 30 I=1,N
C**       F(I)=SQRT(1.0-(T(I)*T(I)))
          F(I)=1.0
30        CONTINUE
          DO 40 I=1,N
          SINEB=(D*SIND(THH(I)))/R
          B=ASIN(SINEB)
          X1=-1.0*R*COS(B)
          X2=+1.0*R*COS(B)
          TH=THH(I)
          CALL INT(FF,X1,X2,I1)
          DAT(I)=I1
C**       TYPE *,I1
40        CONTINUE
          L=1
          DO 50 K=L,N
                  DO 50 J=L,N
                  S1=SIND(THH(J+1))
                  S1=S1*S1
                  S2=SIND(THH(J))
```

```
                  S2=S2*S2
                  S3=SIND(THH(K))
                  S3=S3*S3
                  Q1=S1-S3
                  Q2=S2-S3
                  Q1=SQRT(Q1)
                  Q2=SQRT(Q2)
                  Q=Q1-Q2
                  S(K,J)=2.0*D*Q
50                CONTINUE
                  L=L+1
60          CONTINUE
            FX(N)=DAT(N)/S(N,N)
            I=N-1
70          SUM=0.0
            DO 80 J=I+1,N
            SUM=SUM+(S(I,J)*FX(J))
80          CONTINUE
            FX(I)=(DAT(I)-SUM)/S(I,I)
            I=I-1
            IF(I.NE.0)GO TO 70
            DO 81 J=1,N+1
            AREA(J)=AREA(J)*FX(J)
            ERR(J)=F(J)-FX(J)
81          CONTINUE
            SUMM2=0.0
            DO 82 I=1,N
            SUMM2=SUMM2+AREA(I)
82          CONTINUE
            AVER=SUMM2/SUMM1
            TYPE *,AVER
            WRITE(21,100)R
100         FORMAT(1H ///////////////////////1X,3X,10HRADIUS  R=,F12.8/)
            WRITE(21,110)D
110         FORMAT(1X,3X,22HDISTANCE OF SOURCE  D=,F12.8/)
            WRITE(21,120)THH(1)
120         FORMAT(1X,3X,20HSTARTING  ANGLE  TH=,F12.8/)
            WRITE(21,130)DTH
130         FORMAT(1X,3X,19HSTEP OF ANGLE  DTH=,F12.8/)
            WRITE(21,140)E
140         FORMAT(1X,3X,24HERROR OF INTEGRATION  E=,F12.8/)
            WRITE(21,150)N
150         FORMAT(1X,3X,27HNUMBER OF ANNULUS RINGS  N=,I6/)
            WRITE(21,160)
160         FORMAT(1X,20X,'TABLE:DISTRIBUTION-F(r)= 1.000 '/)
            WRITE(21,170)
170         FORMAT(1X,'______________________________________________________________
           1_________________________________________')
            WRITE(21,180)
180         FORMAT(1X,5X,'TH',9X,'DATA',13X,'r',11X,'F(r)',12X,'FC(r)',8
           1RROR')
            WRITE(21,190)
190         FORMAT(1X,'______________________________________________________________
           1_________________________________________')
            DO 200 I=1,N
            WRITE(21,195)THH(I),DAT(I),T(I),F(I),FX(I),ERR(I)
195         FORMAT(1X,3X,F5.2,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.
200         CONTINUE
            WRITE(21,210)AVER
210         FORMAT(1X,//////20X,'AVERAGE DISTRIBUTION ='F10.8)
            STOP
```

```
C*****************************************
      SUBROUTINE INT(F,X1,X2,I1)
      REAL I0,I1
      COMMON TH,D,E
      H=(X2-X1)/2.0
      L=2
      X=X1
      CALL FUNC(F,X,TH,D)
      F1=F
      X=X2
      CALL FUNC(F,X,TH,D)
      F2=F
      S1=F1+F2
      S2=0.0
      X=X1+H
      CALL FUNC(F,X,TH,D)
      S4=F
      I0=0.0
      I1=(S1+4.0*S4)*(H/3.0)
      IF(I1.EQ.0.0)GO TO 6
10    IF(ABS((I1-I0)/I1).LE.E)GO TO 6
      S2=S2+S4
      S4=0.0
      X=X1+(H/2.0)

      DO 2 J=1,L
      CALL FUNC(F,X,TH,D)
      S4=S4+F
      X=X+H
2     CONTINUE
      H=H/2.0
      L=2*L
      I0=I1
      I1=(S1+2.0*S2+4.0*S4)*(H/3.0)
      GO TO 10
6     RETURN
      END
C*****************************************
      SUBROUTINE FUNC(F,X,TH,D)
      REAL F
      P1=D*SIND(TH)
      P2=P1*P1
      P3=X*X
      P4=P2+P3
      FA=SQRT(P4)
      F=1.0
      RETURN
      END
```

```
C**       THIS PROGRAM RECONSTRUCTS THE DISTRIBUTION
C**
          INTEGER N,CODE,N1,N2,N3,M
          REAL L0,I1,DAT(15),THH(15),S(15,15),F(15),FX(15)
          REAL T(15),DA(15),D3(15),ER(15)
          REAL DC(15),D,TEMP,D1,D2,D3
          COMMON TH,D,E,CODE
          READ(20,*)R,D,E,N,N1,N2,N3
          T(1)=0.0
          DO 1 I=2,N+1
          T(I)=T(I-1)+0.1
1         CONTINUE
          THH(1)=0.0
          DO 10 I=2,N+1
          SINTH=T(I)/D
          AN=ASIN(SINTH)
          THH(I)=(180.0*AN)/3.1415927
10        CONTINUE
          R1=0.4
          R2=0.8
          R3=1.0
          DO 11 I=1,N1
          F(I)=0.0
11        CONTINUE
          DO 12 I=N1+1,N2+N1
          F(I)=1.0
12        CONTINUE
          DO 13 I=N1+N2+1,N
          F(I)=2.0
13        CONTINUE
          DO 40 I=1,N
          TH=THH(I)
          M=I
          SINB1=(D*SIND(THH(I)))/R1
          B1=ASIN(SINB1)
          SINB2=(D*SIND(THH(I)))/R2
          B2=ASIN(SINB2)
          SINB3=(D*SIND(THH(I)))/R3
          B3=ASIN(SINB3)
          IF(M.GT.N1)GO TO 51
          X1=0.0
          X2=+1.0*R1*COS(B1)
          CODE=1
          CALL INT(FF,X1,X2,I1)
          D1=2.0*I1
          CODE=2
          X1=1.0*R1*COS(B1)
          X2=1.0*R2*COS(B2)
          CALL INT(FF,X1,X2,I1)
          D2=2.0*I1
          CODE=3
          X1=1.0*R2*COS(B2)
          X2=1.0*R3*COS(B3)
          CALL INT(FF,X1,X2,I1)
          D3=2.0*I1
          GO TO 59
51        IF(M.GT.(N1+N2))GO TO 52
          CODE=2
          X1=0.0
          X2=1.0*R2*COS(B2)
```

```
      CALL INT(FF,X1,X2,I1)
      D2=2.0*I1
      CODE=3
      X1=1.0*R2*COS(B2)
      X2=1.0*R3*COS(B3)
      CALL INT(FF,X1,X2,I1)
      D3=2.0*I1
      GO TO 59
52    CODE=3
      X1=0.0
      X2=1.0*R3*COS(B3)
      CALL INT(FF,X1,X2,I1)
      D3=2.0*I1
      GO TO 59
59    IF(M.LE.N1)GO TO 35
      IF(M.LE.(N1+N2))GO TO 36
      DAT(M)=D3
      GO TO 39
35    DAT(M)=D1+D2+D3
      GO TO 39
36    DAT(M)=D2+D3
      GO TO 39
39    M=I
40    CONTINUE
      L=1
      DO 60 K=L,N
            DO 50 J=L,N
            S1=SIND(THH(J+1))
            S1=S1*S1
            S2=SIND(THH(J))
            S2=S2*S2
            S3=SIND(THH(K))
            S3=S3*S3
            Q1=S1-S3
            Q2=S2-S3
            Q1=SQRT(Q1)
            Q2=SQRT(Q2)
            Q=Q1-Q2
            S(K,J)=2.0*D*Q
50          CONTINUE
            L=L+1
60    CONTINUE
      FX(N)=DAT(N)/S(N,N)
      I=N-1
70    SUM=0.0
      DO 80 J=I+1,N
      SUM=SUM+(S(I,J)*FX(J))
80    CONTINUE
      FX(I)=(DAT(I)-SUM)/S(I,I)
      I=I-1
      IF(I.NE.0)GO TO 70
      DO 90 I=1,N
      ER(I)=F(I)-FX(I)
90    CONTINUE
      WRITE(21,100)R
100   FORMAT(1X,3X,10HRADIUS   R=,F12.8)
      WRITE(21,110)D
110   FORMAT(1X,3X,22HDISTANCE OF SOURCE   D=,F12.8)
```

```
      WRITE(21,120)THH(1)
120   FORMAT(1X,3X,20HSTARTING  ANGLE   TH=,F12.8)
      WRITE(21,130)DTH
130   FORMAT(1X,3X,19HSTEP OF ANGLE   DTH=,F12.8)
      WRITE(21,140)E
140   FORMAT(1X,3X,24HERROR OF INTEGRATION   E=,F12.8)
      WRITE(21,150)N
150   FORMAT(1X,3X,27HNUMBER OF ANNULUS RINGS   N=,I4)
      WRITE(21,160)
160   FORMAT(1X,20X,'TABLE:DISTRIBUTION-F(r)=        ')
      WRITE(21,170)
170   FORMAT(1X,'--------------------------------------------------------------
      WRITE(21,180)
180   FORMAT(1X,5X,'TH',9X,'DATA',13X,'r',11X,'F(r)',12X,'FC(r)')
      WRITE(21,190)
190   FORMAT(1X,'-------------------------------------------------------------'
      DO 200 I=1,N
      WRITE(21,195)THH(I),DAT(I),R(I),F(I),FX(I),ER(I)
195   FORMAT(1X,3X,F5.2,4X,F11.6,4X,F11.8,4X,F11.8,4X,F11.8,4X,F1
200   CONTINUE
      STOP
      END


C*************************************************
      SUBROUTINE INT(F,X1,X2,I1)
      REAL I0,I1
      COMMON TH,D,E,CODE
      H=(X2-X1)/2.0
      L=2
      X=X1
      CALL FUNC(F,X,TH,D)
      F1=F
      X=X2
      CALL FUNC(F,X,TH,D)
      F2=F
      S1=F1+F2
      S2=0.0
      X=X1+H
      CALL FUNC(F,X,TH,D)
      S4=F
      I0=0.0
      I1=(S1+4.0*S4)*(H/3.0)
      IF(I1.EQ.0.0)GO TO 5
10    IF(ABS((I1-I0)/I1).LE.E)GO TO 6
      S2=S2+S4
      S4=0.0
      X=X1+(H/2.0)

      DO 2 J=1,L
      CALL FUNC(F,X,TH,D)
      S4=S4+F
      X=X+H
2     CONTINUE
      H=H/2.0
      L=2*L
      I0=I1
      I1=(S1+2.0*S2+4.0*S4)*(H/3.0)
      GO TO 10
5     RETURN
```

```
      END
C************************************************
      SUBROUTINE FUNC(F,X,II,J)
      INTEGER CODE
      REAL F
      COMMON AA,BB,CC,CODE
      GO TO (10,20,30),CODE
10    CONTINUE
      F=0.0
      GO TO 50
20    CONTINUE
      F=1.0
      GO TO 50
30    CONTINUE
      F=2.0
      GO TO 50
50    RETURN
      END
```

```
C**       THIS PROGRAM CALCULATES CPMS FOR ALL SCANS
C**       1 TO 5 ,IT READS DATA FROM DATA FILE IS
C**       FOR20.DAT AND THE OUTPUT FILE IS
C**       FOR21.DAT

C**

          INTEGER N,CODE,N1,N2,N3,M
          REAL DAT(15),THH(15),S(15,15),F(15),FK(15)
          REAL DB(15),AREA(15),V,T(15)
          COMMON TH,D,F,CODE
          READ(20,*)R,D,TH,DTH,N,V
          N=N+1
          READ(20,*)(DB(I),I=1,N)
          READ(20,*)(DAT(I),I=1,N)
           DO 1 I=1,N
          TYPE *,DB(I)
1         CONTINUE


            DO 10 I=1,N
          TEMP=DAT(I)
         DAT(I)=ALOG(TEMP)
          TYPE *,DAT(I)
10        CONTINUE
          THH(1)=0.0
          THH(N+1)=ASIN(R/D)
          THH(N+1)=(180.0*THH(N+1))/3.1415927
          DO 20 I=2,N+1
          THH(I)=THH(I-1)+DTH
20        CONTINUE
          DO 30 I=1,N
          F(I)=V
30        CONTINUE
          DO 31 I=1,N
          T(I)=(D*SIND(THH(I)))
31        CONTINUE
          DO 39 I=1,N
          AREA(I)=3.1415927*((T(I+1)**2)-(T(I)**2))
39        CONTINUE


          L=1
          DO 50 K=L,N
                    DO 50 J=L,N
                    S1=SIND(THH(J+1))
                    S1=S1*S1
                    S2=SIND(THH(J))
                    S2=S2*S2
                    S3=SIND(THH(K))
                    S3=S3*S3
                    Q1=S1-S3
                    Q2=S2-S3
                    Q1=SQRT(Q1)
                    Q2=SQRT(Q2)
                    Q=Q1-Q2
                    S(K,J)=2.0*D*Q
50                  CONTINUE
                    L=L+1
50        CONTINUE
          FK(N)=DAT(N)/S(N,N)
          I=N-1
```

```
70        SUM=0.0
          DO 80 J=I+1,N
          SUM=SUM+(S(I,J)*FX(J))
80        CONTINUE
          FX(I)=(DAT(I)-SUM)/S(I,I)
          I=I-1
          IF(I.NE.0)GO TO 70
          DO 91 K=1,N
          TEMP=FX(K)-DB(K)
                FX(K)=-TEMP
91        CONTINUE
          DO 99 I=1,N
          TEMP=FX(I)*AREA(I)
          AREA(I)=TEMP
99        CONTINUE
          SUMM1=0.0
          DO 101 I=1,N-3
          SUMM1=SUMM1+AREA(I)
101       CONTINUE
          SUMM2=3.1415927*T(N-2)*T(N-2)
          TYPE *,SUMM2
          AV=SUMM1/SUMM2
          WRITE(21,100)R
100       FORMAT(1X,3X,10HRADIUS   R=,F12.8/)
          WRITE(21,110)D
110       FORMAT(1X,3X,22HDISTANCE OF SOURCE   D=,F12.8/)
          WRITE(21,120)THH(1)
120       FORMAT(1X,3X,20HSTARTING  ANGLE   TH=,F12.8/)
          WRITE(21,130)DTH
130       FORMAT(1X,3X,19HSTEP OF ANGLE   DTH=,F12.8/)
C**             WRITE(21,140)E
C**40     FORMAT(1X,3X,24HERROR OF INTEGRATION   E=,F12.8)
          N=N-1
          WRITE(21,150)N
150       FORMAT(1X,3X,27HNUMBER OF ANNULUS RINGS   N=,I4/)
          N=N+1
          WRITE(21,160)F(5)
160       FORMAT(1X,20X,'TABLE:DISTRIBUTION-F(r)=',F5.3/)
          WRITE(21,170)
170       FORMAT(1X,'-------------------------------------------------------------------------
          1--------------------------------------------------------'/)
          WRITE(21,180)
180       FORMAT(1X,5X,'TH',9X,'DATA',13X,'r',11X,'F(r)',12X,'FC(r)',8X
          1REA PRODUCT'/)
          WRITE(21,190)
190       FORMAT(1X,'-------------------------------------------------------------------------
          1--------------------------------------------------------'/)
          DO 200 I=1,N-2
          WRITE(21,195)THH(I),DAT(I),T(I),F(I),FX(I),AREA(I)
195       FORMAT(1X,3X,F5.2,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.
200       CONTINUE
          WRITE(21,210)AV
210       FORMAT(1X,//////1X,25X,'AVERAGE  ='F14.8)
          STOP
          END
```

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